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# Probabilistic Structural Analysis Methods (PSAM) for Select Space Propulsion System Components

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## TABLE OF CONTENTS

	<b>Page</b>
<b>1. INTRODUCTION .....</b>	<b>1</b>
1.1 Overview of PSAM Program .....	1
1.2 Report Outline .....	3
1.3 Program Objectives .....	4
1.4 PSAM Accomplishments .....	4
<b>2. OVERVIEW OF THE NESSUS PROBABILISTIC STRUCTURAL ANALYSIS COMPUTER PROGRAM .....</b>	<b>6</b>
2.1 Introduction .....	6
2.2 NESSUS Capabilities Overview .....	7
2.3 Structural Model Definition .....	8
2.4 Probabilistic Response and Resistance .....	9
2.5 Component Reliability and Risk .....	9
2.6 System Reliability .....	11
2.7 Code Organization .....	12
2.8 NESSUS Code Documentation .....	15
2.9 Application Problems .....	16
<b>3. PROBABILISTIC ANALYSIS METHODOLOGY .....</b>	<b>29</b>
3.1 Introduction .....	29
3.2 Basic Definitions and Fast Probability Integration (FPI) Concept .....	29
3.2.1 Response, Performance, and Limit-state Function .....	29
3.2.2 Probability Integration .....	30
3.2.3 Most Probable Point Concept .....	30
3.2.4 Distribution Transformation .....	31
3.2.5 Most Probable Point (MPP) Search Procedure .....	33
3.2.6 Probability Sensitivity Analysis .....	34
3.3 First and Second-order .....	37
3.4 Advanced First-order Reliability Method .....	38
3.5 Fast Convolution Method .....	41
3.6 Standard Monte Carlo Method .....	44
3.7 Radius-based Importance Sampling .....	45
3.8 Adaptive Importance Sampling (AIS1 and AIS2) .....	46
3.9 Advanced Mean Value Methods, (MV, AMV, AMV+) .....	53
3.10 System Reliability Analysis (Fault Tree Method) .....	59
<b>4. NESSUS APPLICATION STUDIES AT ROCKETDYNE .....</b>	<b>64</b>
4.1 Summary .....	64
4.2 Major Objectives of Rocketdyne's Effort .....	65

## TABLE OF CONTENTS (Continued)

	<b>Page</b>
4.3 Space Shuttle Main Engine (SSME) Hardware Analysis .....	65
4.3.1 High Pressure Fuel Turbo Pump (HPFTP) Second Stage Turbine Blade-static Linear Analysis .....	65
4.3.2 High Pressure Fuel Turbo Pump (FPFTP) Second Stage Turbine Blade - A Proposed Design - Modal Analysis .....	66
4.3.3 High Pressure Oxidizer Turbopump (HPOTP) Discharge Duct .....	67
4.3.4 SSME Main Injector Row 13 and Row 12 Injector Elements (LOX Post) - Static Non-linear and Fatigue Damage Analysis .....	68
4.3.5 SSME Transfer Tube Line (Two-duct) - Static Linear Buckling Analysis .....	70
4.4 Related Technology Applications .....	70
4.4.1 Structural Mechanics Applications .....	71
4.4.1.1 SSME High Pressure Oxidizer Turbopump (HPOTP) Bearing Cartridge - Probabilistic Fatigue Damage Computation Due to Transient Resonance .....	71
4.4.1.2 SSME Main Combustion Chamber (MCC) Liner-Non- Hot Spot-Probabilistic Low Cycle Fatigue (LCF) Damage Assessment .....	72
4.4.1.3 SSME Heat Exchanger Turn Around Vane - Probabilistic High Cycle Fatigue Damage Reliability Assessment .....	73
4.4.1.4 Probabilistic Assessment of Competing MCC Liner Designs .....	75
4.4.1.5 Space Station Battery Weld Analysis - Taguchi and Probabilistic Analysis .....	76
4.4.2 Non-structural Mechanics Applications .....	77
4.4.2.1 Probabilistic Design and Analysis Applied to Thermal Loading Environments and Thermal Responses .....	77
4.4.2.2 A Probabilistic Casting Porosity Prediction - Methodology .....	78
4.4.2.3 Turbopump Preliminary Design Using Probabilistic Analysis and Robust Design Concepts .....	78
4.4.3 General Concept and Review Papers .....	78
4.5 Concluding Remarks .....	80
 5. NESSUS CODE DOCUMENTATION .....	83
5.1 Getting Started .....	83
5.2 NESSUS/PBEM User's Manual .....	83
5.3 NESSUS/FPI User's Manual .....	83
5.4 NESSUS/FPI Theoretical Manual .....	83
5.5 NESSUS/SRA User's Manual .....	84
5.6 NESSUS/RISK User's Manual .....	84
5.7 NESSUS/SIMFEM User's Manual .....	84
5.8 UTILITIES .....	85
5.8.1 PREFPI .....	85
5.8.2 P/NESSUS .....	85
5.8.3 NESSUS/SHELL User's Manual .....	86

## TABLE OF CONTENTS (Continued)

	<b>Page</b>
5.8.4 DECODE and ENCODE .....	86
5.8.5 CHANGE.PARAMETER .....	86
5.9 Programmer's Manual .....	86
5.10 NESSUS/FEM User's Manual .....	86
5.11 NESSUS/PRE User's Manual .....	87
5.12 NESSUS/LEVEL1 User's Manual .....	87
5.13 NESSUS/PBEM User's Manual .....	87
5.14 NESSUS/SYSTEM and /SYSRSK User's Manual (Alternate System Reliability) .....	87
6. SELECTED PSAM PUBLICATIONS .....	88
7. SUMMARY AND RECOMMENDATIONS .....	96

## LIST OF FIGURES

<b>Figure</b>		<b>Page</b>
1.1	The Probabilistic Structural Analysis Problem .....	2
1.2	Concept of Probabilistic Structural Analysis .....	2
2.1	Structural Risk Assessment Code NESSUS .....	6
2.2	Chronological Summary of NESSUS Capabilities .....	7
2.3	NESSUS Hierarchical Arrangement of Capabilities .....	8
2.4	Probabilistic Fault Tree Analysis Procedure .....	12
2.5	Overview of NESSUS Code Organization .....	13
2.6	Probabilistic Fracture Mechanics Analysis .....	14
2.7	CDF of Root Stress of a Cantilever Plate .....	17
2.8	CDF of Radial Stress of Elastic-Plastic Thick-Walled Cylinder .....	18
2.9	CDF of the Number of Cycles to Failure for the Three Point Bend .....	20
2.10	CDF of Service Life of a Turbine Blade Subject to Creep Rupture .....	21
2.11	Model of Structural System Failure and System Reliability Results .....	23
2.12	Global/Local Approach and Results .....	24
2.13	Probabilistic Fracture Mechanics Analysis .....	25
3.1	Probabilistic Fault Tree Approach for System Reliability Analysis .....	31
3.2	The Definition of Sensitivity Factors .....	36
3.3	First Order Reliability Method .....	38
3.4	Illustration of the Fast Convolution Procedure .....	43
3.5	Reliability Analysis by Monte Carlo Simulation .....	44
3.6	Radius-based Importance Sampling Method .....	45
3.7	Illustration of the AIS Concept .....	47
3.8	Curvature-based AIS Method .....	48

## LIST OF FIGURES (Continued)

<b>Figure</b>		<b>Page</b>
3.9	AIS for System Reliability .....	51
3.10	Illustration of the AMV Method .....	56
3.11	AMV+ Iteration Algorithm for a Specified P-level .....	57
3.12	AMV+ Iteration Algorithm for a Specified Z-level .....	57
3.13	The AMV+ Analysis for the Cantilever Beam Example .....	59
3.14	Probabilistic Fault Tree Approach for System Reliability Analysis .....	60



## **1. INTRODUCTION**

### **1.1 Overview of PSAM Program**

Probabilistic Structural Analysis Methods (PSAM) for Select Space Propulsion System Components is a research and technology program sponsored by the National Aeronautics and Space Administration - Lewis Research Center (NASA-LeRC). The objective of PSAM was to develop analysis methods and computer programs for the probabilistic structural analysis of engine components for current and future space propulsion systems. It was envisioned that these methods and computational tools would play a critical role in establishing increased system performance and durability and assist in structural system qualification and certification.

An example problem addressed by the PSAM program is depicted in Figure 1.1. The structure, illustrated by a SSME turbine blade, is subjected to stochastic thermomechanical launch loads. Uncertainties or randomness may also occur in material properties, structural geometry, and boundary conditions. Material property stochasticity, such as in modulus of elasticity or yield strength, exists in every structure and is a consequence of variations in material composition and manufacturing processes. Mechanical properties vary significantly over the temperature ranges encountered in SSME operation, and uncertainty in the thermal environment introduces another element of stochasticity into the material description.

Component fabrication, through variations in normal manufacturing dimensions such as thickness, adds another degree of randomness. Critical engine components are manufactured with high precision, and thus, nominal geometrical variations are expected to be small. However, for the turbine blade shown in Figure 1.1, variations in blade twist in the span-wise direction could have a significant effect on the structural response to aerodynamic loads.

Assumptions in boundary conditions are frequently a major source of uncertainty in analysis. Cantilever type structures, such as this turbine blade, are often modeled as fully restrained for simplicity. On the other hand, because of assembly procedures, real structures may exhibit some degree of flexibility at the support. The assumption on boundary conditions may not be important in some types of analyses. However, computations involving the dynamic response of a turbine blade could be very sensitive to variations in boundary conditions.

Assuming that the loading, material properties, structural geometry and boundary conditions are described in a probabilistic sense, the objective of the PSAM is to develop analytical methods and computer programs for computing the probabilistic structural response or reliability of the structural components. The response variable or variables could be static or dynamic deflections, strains, and stresses at one or several locations, natural frequencies, fatigue or creep life, etc. Figure 1.2. illustrates how the PSAM methods and codes can simulate input uncertainties and compute probabilistic response or reliability using a finite element model.

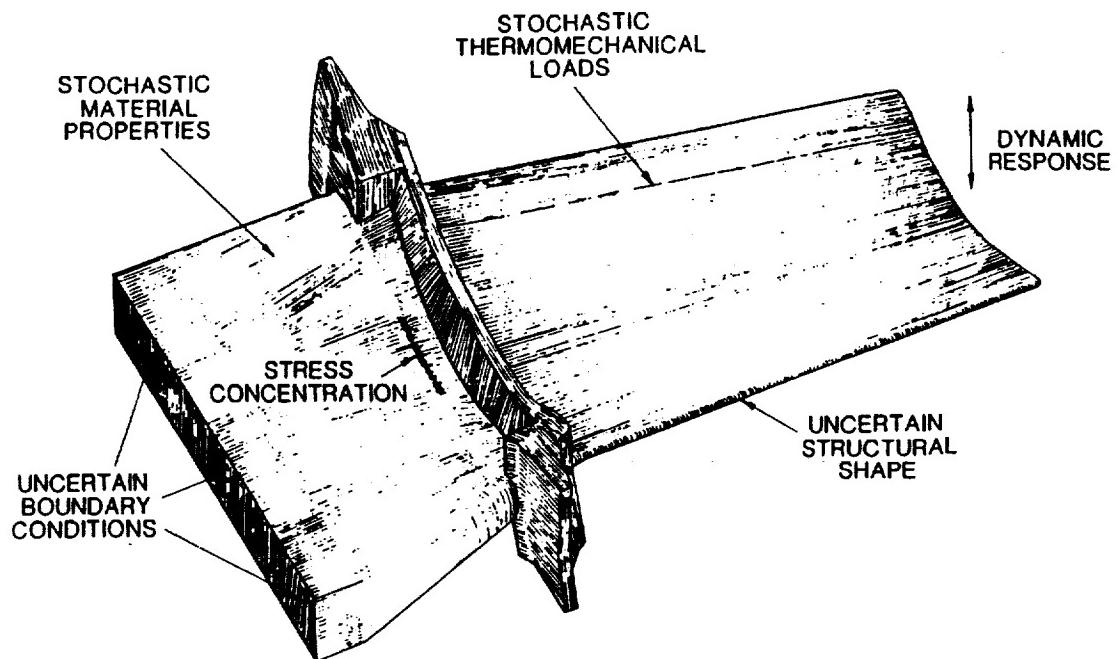


Figure 1.1 - The Probabilistic Structural Analysis Problem

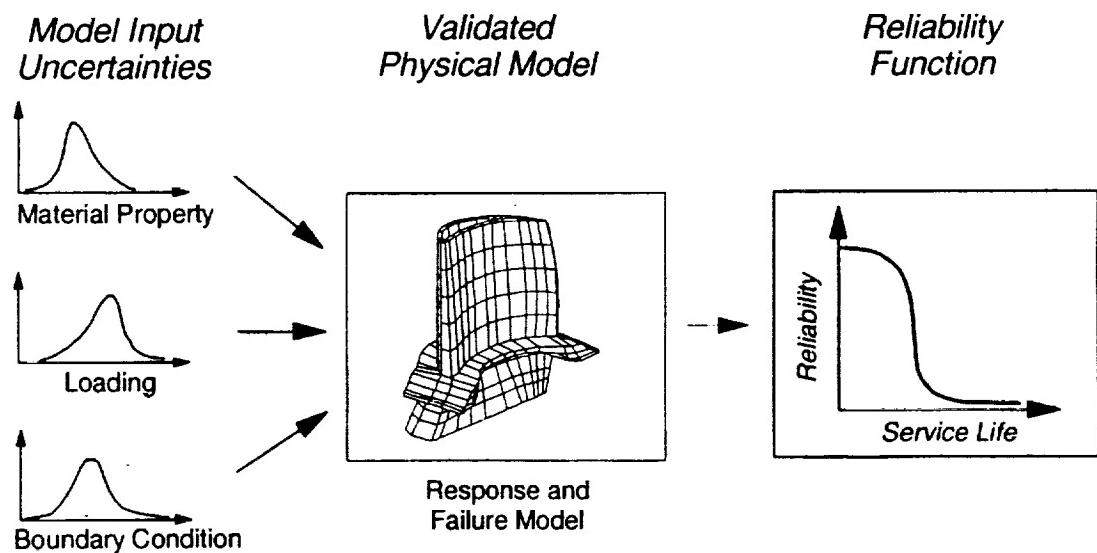


Figure 1.2 - Concept of Probabilistic Structural Analysis

PSAM has two phases. Phase I, from October 1984 to December 1989, completed development work on probabilistic finite elements, probabilistic approximate analysis methods, and advanced probabilistic analysis methods. A probabilistic finite element program NESSUS (Numerical Evaluation of Stochastic Structures Under Stress), Version 4, was completed, which can perform probabilistic structural analyses. Phase II, from January 1990 until September 1993, completed component risk assessment and system risk assessment methodology and computer program development. NESSUS 6.1 was completed which can perform component and system risk assessment. Funding was unavailable in fiscal year 1994 for the planned structural system qualification and certification work. In February of 1995, the PSAM project was officially closed with a final close-out funding to conduct a NESSUS workshop at NASA-LeRC (conducted in July, 1995), to deliver the final NESSUS version (Version 6.2 delivered in September 1995) to NASA, and to write a final summary report.

The prime contractor was Southwest Research Institute (SwRI). Its efforts were supported by: MARC Analysis Research Corporation - NESSUS/FEM probabilistic finite element code development (Phase I); Rocketdyne Division, Rockwell International Corp. - Space Shuttle Main Engine (SSME) design and hardware experience; and several university consultants. There were many people who contributed to this long-term project. These include the following members who made significant long-term contributions:

NASA-LeRC project manager: C. C. Chamis.

SwRI Project Managers: Y.-T. Wu (1991-95), T.A. Cruse (1985-90), O.H. Burnside (1984-85).

SwRI Team Members: H.R. Millwater, B.H. Thacker, D.S. Riha, T. Y. Tornq.

Rocketdyne: K.R. Rajagopal, J.F. Newell.

Marc Analysis Co.: J. B. Dias.

University Consultants: T.A. Cruse, P.H. Wirsching, S. Mahadevan.

In addition, the following members have significantly contributed to the program by providing technical advice, NESSUS application experience, and NESSUS code improvements: D. A. Hopkins (NASA-LeRC), M. H. Rheinfurth (NASA-MSFC), S. S. Pai (NASA-LeRC), M. C. Shiao (NYMA), A. R. Shah (NYMA), G. E. Orient (Rocketdyne), and S. V. Harren (SwRI). In Section 6, there is a list of selected publications where it is evident that there are several others who have contributed to the PSAM program.

## **1.2 Report Outline**

This report summarizes the work accomplished by the PSAM team. There are six sections in this report. Section 1 provides the overview, the objectives, and the accomplishment of the PSAM program. Section 2 gives an overview of the NESSUS program, which is the main product of PSAM. Section 3 provides a summary of the probabilistic analysis methods that

have been implemented in NESSUS. Many of these methods have been developed to address the problems associated with complicated models that require computer-intensive calculations. Section 4 summarizes Rocketdyne's verification/application studies. Section 5 summarizes the over eight hundred pages of NESSUS documentation. Section 6 gives a list of selected PSAM publications with brief descriptions. Section 7 provides a summary and recommendations

### **1.3 Program Objectives**

The purpose of the PSAM project was to develop a new technology capability for the analysis of advanced space propulsion system hardware. Specifically, the focus of the PSAM effort was to develop probabilistic structural analysis capabilities which can simulate/model mechanical and thermal loads, geometry, and material response subject to uncertainty. The PSAM program consisted of several major technical thrusts: probabilistic finite element methods, probabilistic approximate analysis methods, probabilistic advanced analysis methods, component risk assessment, and system risk assessment.

### **1.4 PSAM Accomplishments**

PSAM has successfully integrated computationally-intensive structural analysis methods (finite elements, boundary elements) with advanced reliability methods to simulate uncertainties in loads, materials, geometries, and boundary conditions. NESSUS was the first major probabilistic finite element program that could solve complicated reliability analysis problems such as a turbine blade with multiple failure modes. The probabilistic methods developed specifically for PSAM include the advanced mean value method and the adaptive importance sampling methods which have been proven to be fast and accurate and well suited for a wide range of engineering applications.

The capability of NESSUS includes predicting structural reliability, determining dominant variables affecting reliability, performing probabilistic sensitivity analysis to identify data sensitivity and data needs, assessing the structural risk in terms of cost, and determining optimum inspection intervals. The analyses can be used to provide information for retirement for cause decisions, certify the structure subject to reliability requirements, and support the definition of testing requirements for certification procedures.

In addition to applying the technology to aerospace propulsion structures (including SSME) by Rocketdyne, the NESSUS technology is general and has been applied by SwRI, NASA, and many industries to various engineering systems that include automotive, aircraft, offshore, geomechanics, biomechanics, and other structural systems.

PSAM has made major contributions to the open literature. Section 6 includes a list of sixty selected PSAM-related publications. This list does not include numerous other

publications that applied the PSAM and NESSUS technology but were not supported by the project. The PSAM methods and the NESSUS code have been quoted frequently in the open literature and have stimulated other code developments.

The SwRI team has conducted PSAM technology transfers through its annual short course since 1990 and on-site short courses at industrial companies. More than two hundred engineers have been trained through these courses.

In summary, the PSAM project not only has met it's objective, but has made a major impact on the development of reliability technology and played a major role in promoting the widespread use of probabilistic analysis and design technology.

## 2. OVERVIEW OF THE NESSUS PROBABILISTIC STRUCTURAL ANALYSIS COMPUTER PROGRAM

### 2.1 Introduction

The NESSUS probabilistic structural analysis computer program combines state-of-the-art probabilistic algorithms with general purpose structural analysis methods to compute the probabilistic response and the reliability of engineering structures. Uncertainty in loading, material properties, geometry, boundary conditions and initial conditions can be simulated. The structural analysis methods include nonlinear finite element methods, boundary element methods, and user-written subroutines. Several probabilistic algorithms are available such as the advanced mean value method and the adaptive importance sampling method.

The application of the code includes probabilistic structural response, component and system reliability and risk analysis of structures considering cost of failure. Several application problems are presented which describe some of the capabilities.

Figure 2.1 summarizes the overall capabilities of NESSUS. As seen in the figure, NESSUS contains an integration of probabilistic methods with nonlinear finite element and boundary element methods. A general interface for defining random variables is included. A variety of probabilistic results can be obtained from the analysis of a user-defined structural model.

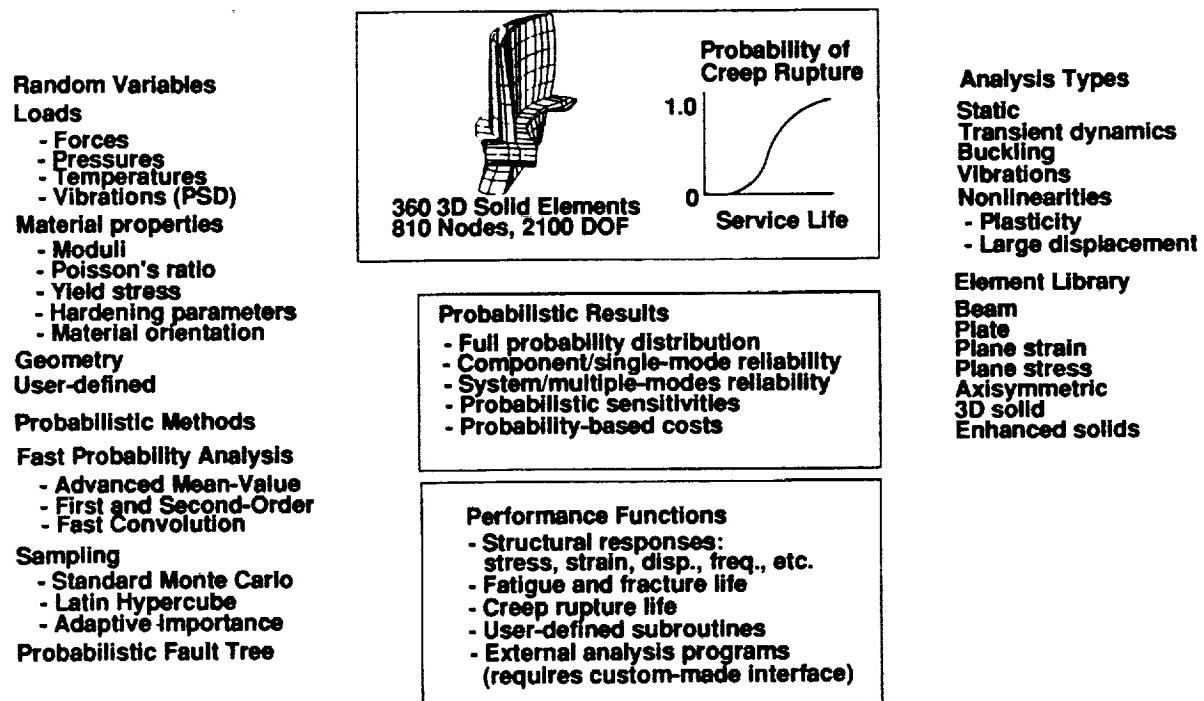


Figure 2.1 - Structural Risk Assessment Code NESSUS

A chronological summary of the new capabilities in the recent versions of NESSUS is presented in Figure 2.2. NESSUS version 4.0, delivered to NASA/LeRC in October 1989, addressed probabilistic structural response analysis. Version 5.0, delivered in March 1991, addressed component reliability, resistance, and risk. Version 5.5, delivered in October 1991, addressed system reliability using a probabilistic fault tree method. Version 6.0, delivered in May 1992, included an alternate system reliability, and a boundary element structural modeling method. Version 6.1, delivered in 1993, enhanced the user-friendliness by including an FPI preprocessor and an equation input option. The final version of NESSUS, Version 6.2, was delivered in September 1995.

NESSUS Code Delivery Status		
Version	Delivery Date	New Capabilities
4.0	October 1989	Probabilistic Structural Response Analysis
5.0	March 1991	Component Reliability, Resistance, and Risk
5.5	October 1991	System Reliability, Structural Sampling
6.0	May 1992	Alternate System Reliability, Probabilistic Boundary Element
6.1	July 1993	FPI Preprocessor, Equation Input
6.2	September 1995	Final Code Delivered

Figure 2.2 - Chronological Summary of NESSUS Capabilities

## **2.2 NESSUS Capabilities Overview**

The NESSUS code is structured in a hierarchical fashion. More recent advanced capabilities build on previous more fundamental capabilities. NESSUS is structured in a modular format such that new algorithms and technology can be quickly implemented and tested. A variety of probabilistic algorithms are implemented.

Figure 2.3 shows a schematic of the capabilities and their relationships. The foundation is a robust, flexible structural modeling capability for defining the analysis model. Advanced probabilistic algorithms combine with the analysis model to determine the cumulative distribution function (CDF) of a structural response or resistance. Component reliability combines the structural response and resistance to determine the reliability of the component. System reliability considers the reliability of a system consisting of multiple failure modes and multiple components. System risk assessment combines system reliability with cost of failure, replacement, inspection, etc. to determine the system risk. System qualification/certification involves assessing the structural system to determine whether the structure can meet the

qualification/certification reliability requirements. Health monitoring involves monitoring the health of the structural system with respect to damage initiation sites, inspection criteria and intervals, and retirement for cause. The current version of NESSUS, version 6.2, contains the capabilities through system reliability and risk. The code organization is given in Section 2.7.

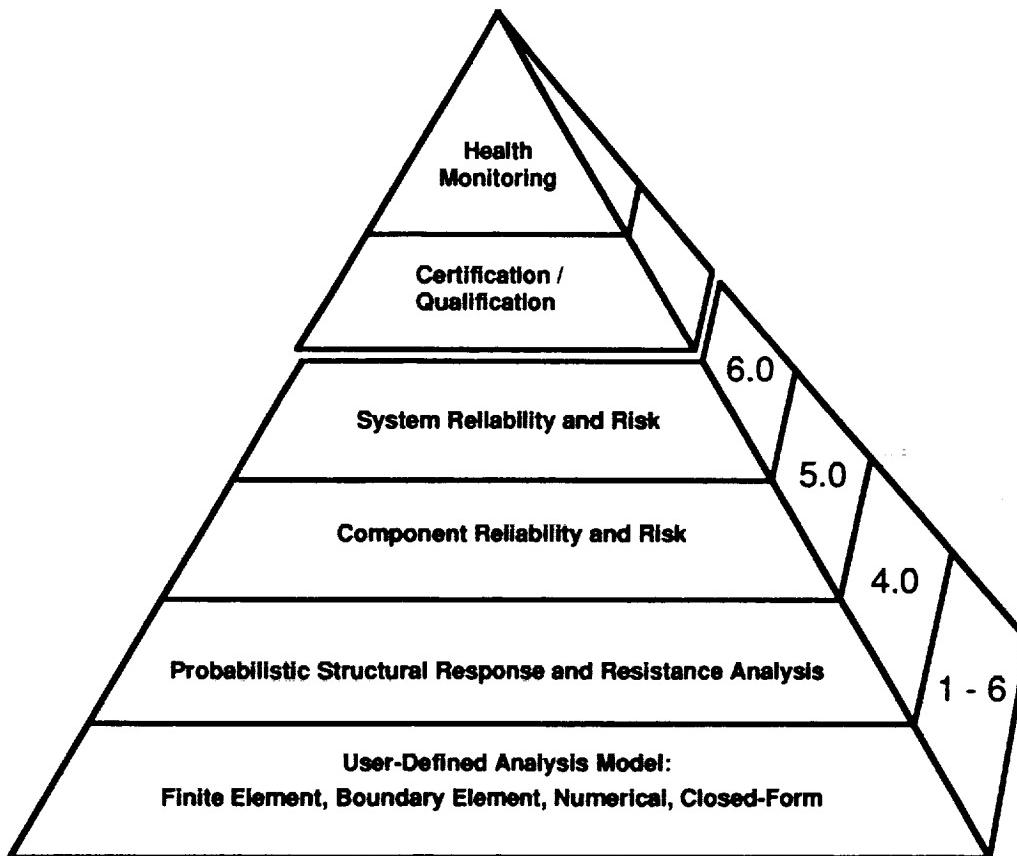


Figure 2.3 - NESSUS Hierarchical Arrangement of Capabilities

### **2.3 Structural Model Definition**

A useful program must allow the user flexibility in defining his or her model to be analyzed. NESSUS provides this capability through a combination of a self-contained finite element/boundary element module and user-written subroutines. The user can define the structural reliability performance function using, the finite/boundary element model, a damage model, material strength or resistance model, and other numerical routines. The finite element

method implemented in NESSUS contains several unique methods for efficient sensitivity analysis [Reference 2.1]. Alternatively, the boundary element method can be used for structures when appropriate [Reference 2.2].

NESSUS contains a flexible interface for defining the structural random variables. The user can define geometric, loading, material properties, boundary conditions, and initial conditions of any part of the structure as random variables. In addition, nonstructural random variables, i.e., those variables not included in defining the structural model, can be included as shown in References 2.3-2.5.

## **2.4 Probabilistic Response and Resistance**

The early focus of NESSUS was on computing the CDF of a structural response such as stress, displacement, natural frequency, etc. in an accurate and efficient manner. Conventional simulation methods are too costly as hundreds of thousands of computations may be required; especially for the tails of the distribution. As a result, the advanced mean value (AMV) method was developed.

The AMV method is a mean-based predictor-corrector method which uses local gradients to project the results for the entire CDF then corrects the prediction with a re-analysis of the structural model at each predicted point. The local gradients are usually, but not necessarily, taken about the mean values of the random variables. For many problems, this approach has shown to give good results with a minimum of computations [Reference 6].

For some nonlinear problems, a single correction may not be sufficient to accurately compute the CDF [Reference 7]. As a result, the AMV method with iterations, (AMV+), which is an extension to the AMV method, was developed. This method involves new gradient computations about the previously predicted results. The new gradients are then used to compute an improved result. The procedure can be repeated until convergence within a user-specified tolerance. The AMV+ method has been automated in NESSUS. The theoretical background for the AMV+ method is presented in References 2.8-2.10.

## **2.5 Component Reliability and Risk**

Component reliability, defined in terms of the strength ( $R$ ) exceeding the stress ( $S$ ) for a single failure mode, location, or component, is computed by defining the structural stress in terms of a finite element model or closed-form expression and strength in a user-defined subroutine. The complement to reliability, failure, is defined as strength ( $R$ ) less than stress ( $S$ ), or in equation form,

$$g = R - S. \quad (2.1)$$

where failure occurs when  $g \leq 0$ .

Note, however, that R and S may be complex models involving other primitive random variables, i.e.  $R(X_i)$ ,  $S(Y_i)$ . For example, the stress may be the von Mises stress from a finite element model containing random loads, material properties, and geometry.

NESSUS computes the probability of strength exceeding stress using computationally efficient probabilistic and structural algorithms in several ways. NESSUS can compute the CDF and probability density function (PDF) of R and S given the statistics of  $X_i$  and  $Y_i$ . If R and S are independent, the probability of failure can then be computed using the equation

$$p_f = \int_0^{\infty} f_S(s) F_R(s) ds \quad (2.2)$$

where  $f_S$  is the PDF of S and  $F_R$  is the CDF of R.

If R and S contain common random variables or are statistically correlated, Equation 2.2 is not accurate and other means must be used. One approach is to compute the reliability directly from Equation 2.1 and bypass the intermediate R and S results. The results will be accurate because the full functional form is used. This is one approach used by NESSUS.

Another approach is to compute the CDF and PDF of R and S, then create a new response surface function at key combinations of random variables called most probable points, MPP's, for R and S. These response surfaces are then combined according to Equation 2.1 and the reliability is then computed. This approach is accurate because a functional form for R and S is used not just the PDF's and CDF's. This last approach is described in Reference 2.20. Thus, through several different approaches, NESSUS can correctly handle problems with common random variables between the stress and the strength.

A component risk module has been added to NESSUS which will compute the risk with respect to cost or a user defined criteria [Reference 2.11]. Risk is defined in terms of the "consequences" or "severity" of failure. For example, different failure modes may have different consequences. Failure by fracture may be more severe than failure by yield. Thus, the risk due to fracture may be significantly higher than that due to yield even if the probabilities of occurring are the same.

Risk with respect to cost is computed using the formula

$$\text{Risk} = C_0(x) + p_f(x) * C(x) + (1 - p_f) * N_C(x) \quad (2.3)$$

where x is any design variable which can be non-random variables or the response in a reliability analysis such as stress, displacement, or cycles to failure,  $C_0$  is the initial cost as a function of x,  $p_f$  is the probability of failure as a function of x and  $N_C(x)$  is the probability of nonfailure as a function of x. Nonfailure may involve costs associated with maintenance and repair procedures. The  $p_f(x)$  curve may be the probability of failure computed by NESSUS. The RISK module is integrated with NESSUS such that the  $p_f$  and risk can be computed in a single analysis.

## **2.6 System Reliability**

System reliability computes the reliability of a system defined in terms of multiple components, failure modes or locations. One approach implemented is a probabilistic fault tree analysis method (PFTA). The system is defined in terms of "AND" and "OR" gates and bottom events. A "PRIORITY AND" gate can be used if there are sequential effects, i.e., a sequence, to the bottom events.

Each bottom event involves a structural model with an associated definition of failure for that bottom event. The reliability or probability of failure of each bottom event is computed as a component reliability problem. In addition to the probability of failure, NESSUS computes a polynomial approximation to the structural model, called a failure function, at the most probable point. The probability of system failure is then computed using an adaptive importance sampling method on the system failure model defined in terms of the bottom event failure functions and the gate logic. Correlation between bottom events is automatically handled through the failure functions. This is significant as many bottom events are likely to be correlated through common random variables such as loading.

There are several advantages to this approach. Because the failure functions are used, not just the probability of failure of each bottom event, the method can account for correlation, e.g., from common random variables, between bottom events. The preexisting NESSUS capabilities for component reliability can be used to compute the reliability and failure function for each bottom event. In addition, importance sampling is typically an order of magnitude or more faster than conventional Monte Carlo. The PFTA procedure samples the failure functions by default. Sampling of the structural models, e.g., finite element model, can be performed to check critical regions.

A schematic of the analysis procedure is presented in Figure 2.4. This procedure has been automated in NESSUS. More details can be found in References 2.12-2.13.

The adaptive importance sampling method (AIS) is a intelligent sampling method which uses knowledge about the problem from analytical results to reduce the sampling region and thus locate the samples only in critical areas. Accurate results can be obtained with several orders of magnitude fewer samples in comparison to conventional Monte Carlo sampling. The theoretical details can be found in References 2.14-2.15.

### PFTA Procedure

- 1) Define the fault tree using "AND" and "OR" gates and "BOTTOM EVENTS."
- 2) Define the bottom events using a finite element model and/or closed-form equations.
- 3) Compute the component reliability and the failure function<sup>\*</sup> (first or second order polynomial) at the most probable point of each bottom event.
- 4) Combine the failure functions according to the fault tree structure.
- 5) Compute the system reliability using an adaptive importance sampling method by sampling,
  - a) the failure functions, or
  - b) the actual bottom events.

\* The failure function is defined as the polynomial approximation to the bottom event at the MPP.

Figure 2.4 - Probabilistic Fault Tree Analysis Procedure

System risk assessment is computed by combining the cost of failure, in terms of replacement, inspection, repair, etc. of individual components of the system with their probability of failure. The result is upper and lower bounds on the cost of operating the system.

## **2.7 Code Organization**

NESSUS is structured in a modular format such that new algorithms and technology can be independently and quickly implemented and tested. The algorithmic modules interact through several "driver" modules. The primary technical modules are two deterministic structural analysis modules, i.e. finite element and boundary element methods, and a probabilistic analysis module, Fast Probability Integration (FPI). The major modules are described below. A schematic of NESSUS is given in Figure 2.5.

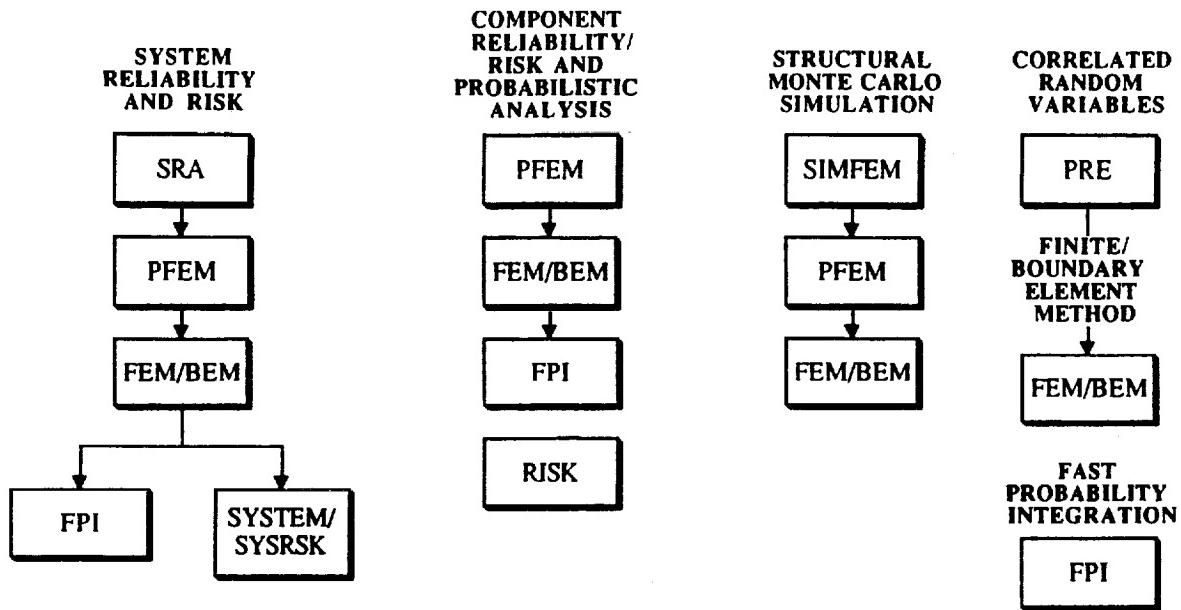


Figure 2.5 - Overview of NESSUS Code Organization

#### SRA - System Risk Assessment

SRA is the driving module for system risk assessment in NESSUS. SRA coordinates the operations between the PFEM and FPI modules. The present capabilities consist of a probabilistic fault tree analysis (PFTA) method.

#### PFEM - Probabilistic Finite Element Module

PFEM is the driving module for component risk assessment in NESSUS. PFEM coordinates the operations between the finite element code and FPI. Essentially, PFEM implements the advanced mean value algorithm with iterations AMV+ [References 2.8-2.10]. PFEM also integrates the structural model with other analysis capabilities defined in user-written subroutines such as fatigue calculations. Thus, the performance can be more general than structural response.

### SIMFEM - Simulation Finite Element Module

SIMFEM is the driving module for conventional Monte Carlo and Latin Hypercube Sampling of the structure, i.e., finite element model.

### FEM - Finite Element Module

The finite element method in NESSUS is a nonlinear static and dynamic code with several advanced features necessary for probabilistic analysis [Reference 2.1]. A summary of the capabilities of the FEM code is presented in Figure 2.6.

<b>ANALYSIS TYPES</b>	<b>ELEMENT LIBRARY</b>	<b>RANDOM VARIABLES</b>
STATIC	BEAM	GEOMETRY
NATURAL FREQUENCY	PLATE	LOADS FORCES
BUCKLING	PLANE STRESS	PRESURES TEMPERATURES
HARMONIC EXCITATION	PLANE STRAIN	MATERIAL PROPERTIES
RANDOM VIBRATION	AXISYMMETRIC	ELASTIC MODULUS POISSON'S RATIO SHEAR MODULUS ORIENTATION ANGLE STRESS-STRAIN CURVE DAMPING DENSITY
TRANSIENT DYNAMICS	3D CONTINUUM ENHANCED ELEMENTS	INITIAL CONDITIONS BOUNDARY CONDITIONS
		NON-FINITE ELEMENT RANDOM VARIABLES

Figure 2.6 - NESSUS Finite Element Capabilities

FEM contains a very flexible interface for defining random variables of the structure which allows the user great freedom in defining the random variables.

A special iterative solution algorithm has been implemented in FEM such the solution to a perturbed problem can be computed at a reduced computational time compared to the unperturbed solution once the unperturbed solution has been computed. This feature facilitates the calculation of structural sensitivities.

### BEM - Boundary Element Module

A boundary element module can be used in place of FEM for the structural analysis [Reference 2.2].

### FPI - Fast Probability Integration

The FPI module performs the probability calculations. FPI contains a number of advanced methods in order to compute accurate results quickly. FPI presently contains FORM/SORM, convolution, advance first order, conventional Monte Carlo, importance sampling, adaptive importance sampling, and advanced Mean Value solution methods. More details concerning FPI can be found in Reference 2.17.

### PRE

The PRE module handles random fields, i.e., spatial correlation. PRE computes a set of independent random variables from the random fields using an eigenvalue decomposition [Reference 2.1].

### RISK

The RISK module computes the risk with respect to cost or a user defined criteria. Risk is defined in terms of the "consequences" or "severity" of failure which is integrated with the probability of failure [Reference 2.11].

### SYSTEM

The SYSTEM module is an alternate system reliability method to probabilistic fault tree analysis that computes the upper and lower bounds of the system reliability, see Section 5.

### SYSRSK

The SYSRSK module computes the system risk by combining the cost of failure, in terms of replacement, inspection, repair, etc. of individual components of the system with their probability of failure. The result is upper and lower bounds on the cost of operating the system, see Section 5.

### SHELL

The SHELL program is an X-windows graphical user interface to help the user prepare a NESSUS input file. The current SHELL program supports NESSUS 5.0.

## **2.8 NESSUS Code Documentation**

The NESSUS code has over eight hundred pages of documentation to assist the user. A summary of the code documentation is described in Section 5.

## **2.9 Application Problems**

A focus of the PSAM program has been the application of the technology. Typically, a new development in technology and coding is demonstrated on an application problem. Rocketdyne has annually applied the NESSUS code to an SSME structure. These analyses are difficult and complex, requiring many man-months of effort. Section 4 summarizes the application problems solved by Rocketdyne. Other application problems are designed to be simpler yet demonstrate the important new features of the code. The application problems below are presented to highlight the capabilities of the code and indicate the evolution of the code's development.

The NESSUS code, released 1989, focused on efficient methods for probabilistic structural response analysis. In this case, the response analyzed was a structural response, computed using the finite element module, such as, stress, strain, displacement, and natural frequency. The AMV method was used to compute the CDF of the structural response efficiently. The AMV method was implemented manually, that is, the user had to coordinate data between the finite element and FPI modules.

Reference 2.6 is a summary paper which demonstrates the application of the AMV method with finite element models to compute the CDF for a variety of mechanics problems. Some of the structural responses analyzed are stress, displacement, and buckling load. Correlated random variables are included. The problems chosen are such that an exact solution is obtainable. This reference demonstrates the accuracy and efficiency of the AMV method. The AMV results were typically very close to the exact solution. This reference also shows the important approach of using the ratio of the exact and finite element solutions at the median as a scale factor for the entire CDF. This approach can often correct modeling error using a simple scaling procedure.

Figure 2.7 shows the NESSUS input model and the NESSUS and analytical CDF of the root stress of a cantilever plate subject to correlated loads.

The AMV+ method, i.e., AMV with iterations, was demonstrated in Reference 2.6 but was typically not required as the AMV result was very good. Reference 2.7 demonstrates the application of NESSUS and the AMV+ method to a problem where AMV iterations are required. The problem is a thick walled cylinder with a perfectly plastic material model. The internal pressure and the yield stress are random variables. A closed-form solution for the stress at any location is available.

This paper is valuable for two reasons. First, it demonstrates the application of NESSUS to a materially nonlinear structure. Second, it demonstrates the AMV+ method and describes when the iterations are required. In summary, iterations will be required to obtain a good solution if the sensitivities of the response with respect to the random variables are changing over the range of the distribution. Figure 2.8 shows the input model and the CDF's computed with NESSUS and the analytical solution.

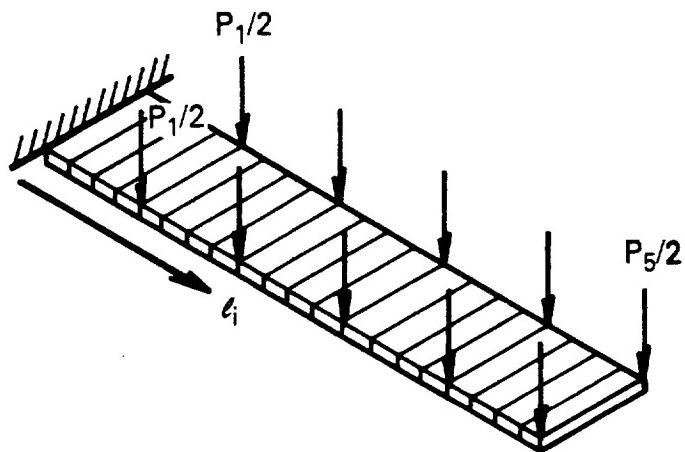
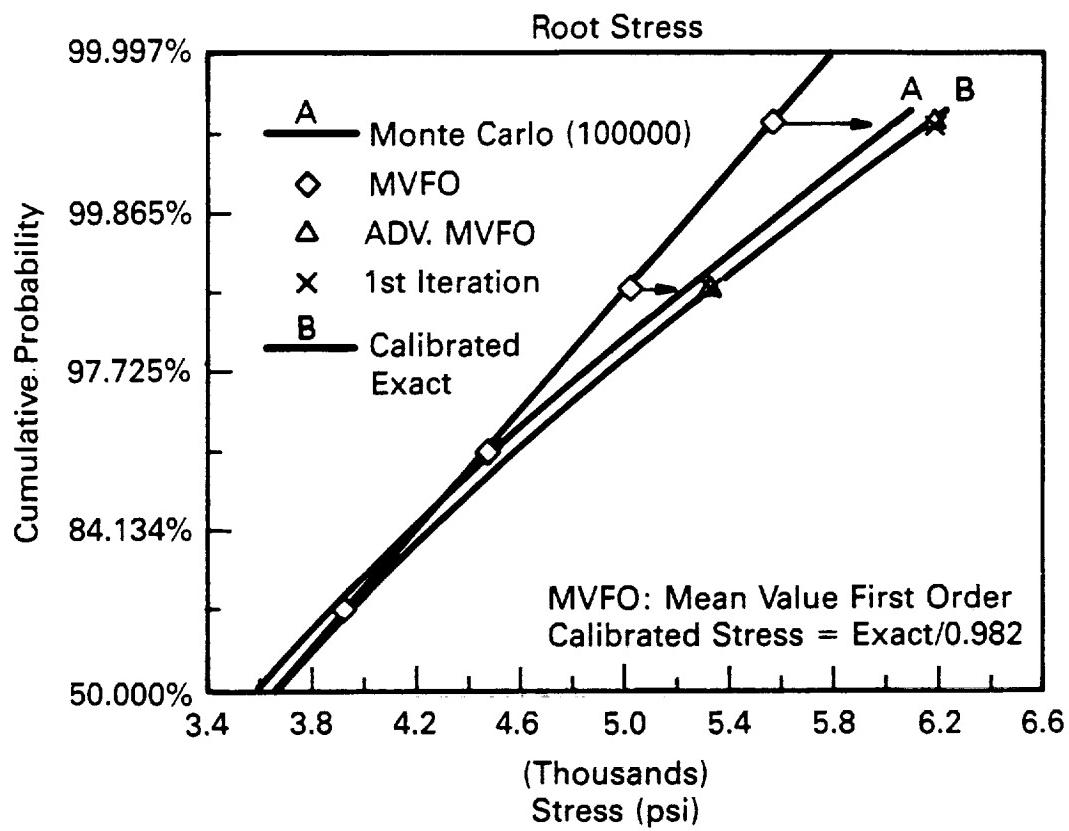


Figure 2.7 - CDF of Root Stress of a Cantilever Plate

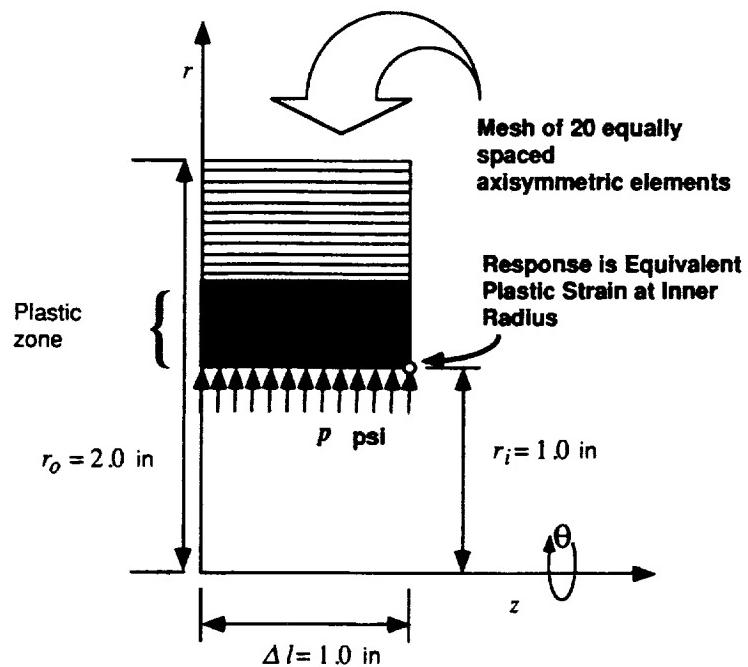
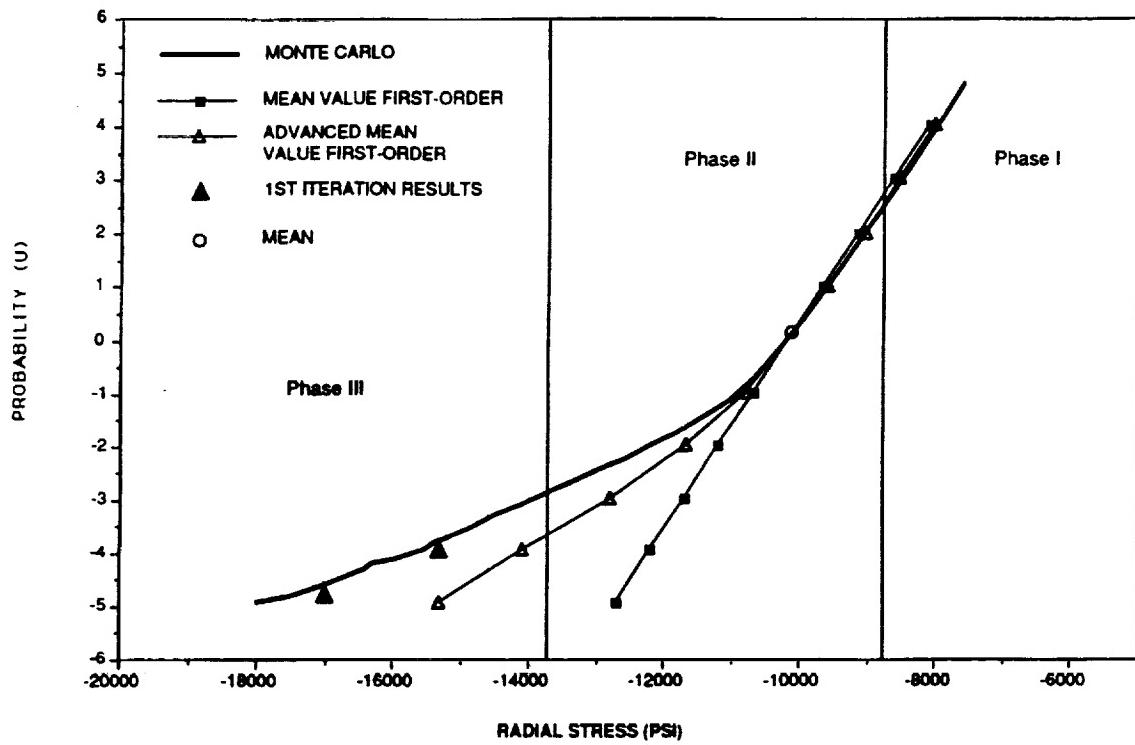


Figure 2.8 - CDF of Radial Stress of Elastic-Plastic Thick-Walled Cylinder

A generalized AMV method which can handle nonmonotonic response functions was developed and demonstrated on several example problems in Reference 2.8. It was demonstrated how AMV can detect the multiple most probable point problem and correct the solution. This methodology has not been automated in NESSUS but the concepts and theoretical basis have been demonstrated.

In 1991, NESSUS, version 5.0, was extended to allow a user-definable response and to automate the complete AMV+ method for both "z" levels, i.e., given a response, compute the probability, and "p" levels, i.e., given a probability, compute a response. These were significant steps in the accuracy, flexibility and user-friendliness of the code.

Allowing a user-definable response greatly enhances the applicability of the code. This feature provides a means to incorporate material resistance, i.e., "strength" with structural response, i.e., "stress". The response to be analyzed is now completely general. It can contain combinations of finite element results, closed-form equations, and other numerical routines. Thus, with this version, it is also possible to analyze life prediction. The response can now be cycles to failure defined by finite element results, closed-form equations and numerical routines.

NESSUS makes a distinction between "computational" and "explicit" random variables. Computational random variables are those that effect the finite element model and explicit random variables are those that do not. Different computational approaches are used for each to maximize efficiency.

Reference 2.18 describes the new features of the code and two application problems. Problem 1 computes the fatigue life of a beam under three point bending using linear elastic fracture mechanics. Problem 2 computes the life of a bar using elastoplastic analysis and a low cycle fatigue model. The bar has a bilinear stress-strain curve and the initial yield stress is degraded as a function of the thermal loading and number of cycles on the bar. Figure 2.9 shows the input model, the failure model and the CDF of the cycles to failure of the three point bend problem.

Probabilistic analysis of structures with random stress-strain material properties is discussed in Reference 2.19. This paper describes an approach to modeling the stress-strain curve as random using a cubic polynomial. Certain restrictions were found between the polynomial coefficients.

Reference 2.20 describes a new capability added to NESSUS for computing structural reliability by post-processing previously computed probabilistic results for stress and strength. The advantage of this approach is that the distributions for stress and strength can be computed independently then combined at a later date. The new approach can correctly account for correlation between the stress and strength due to common random variables. Figure 2.10 shows the stress and strength distributions computed from a creep analysis of a turbine blade. The

reliability is computed for any time by using the previously computed stress and strength distributions. The procedure is very quick and does not require any finite element solutions once the stress distribution has been computed.

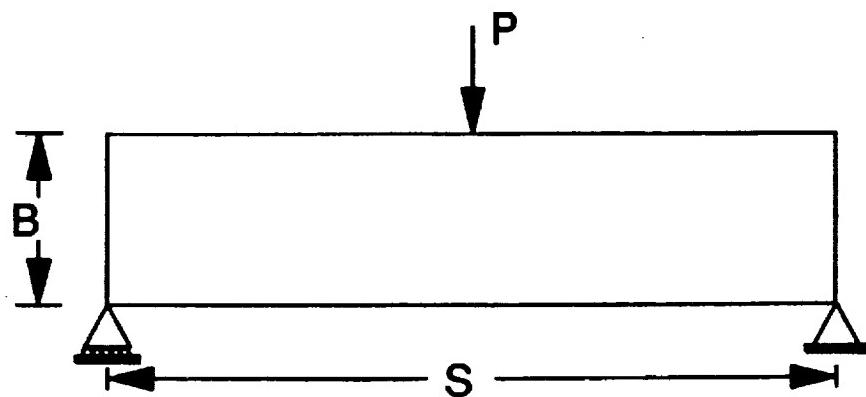
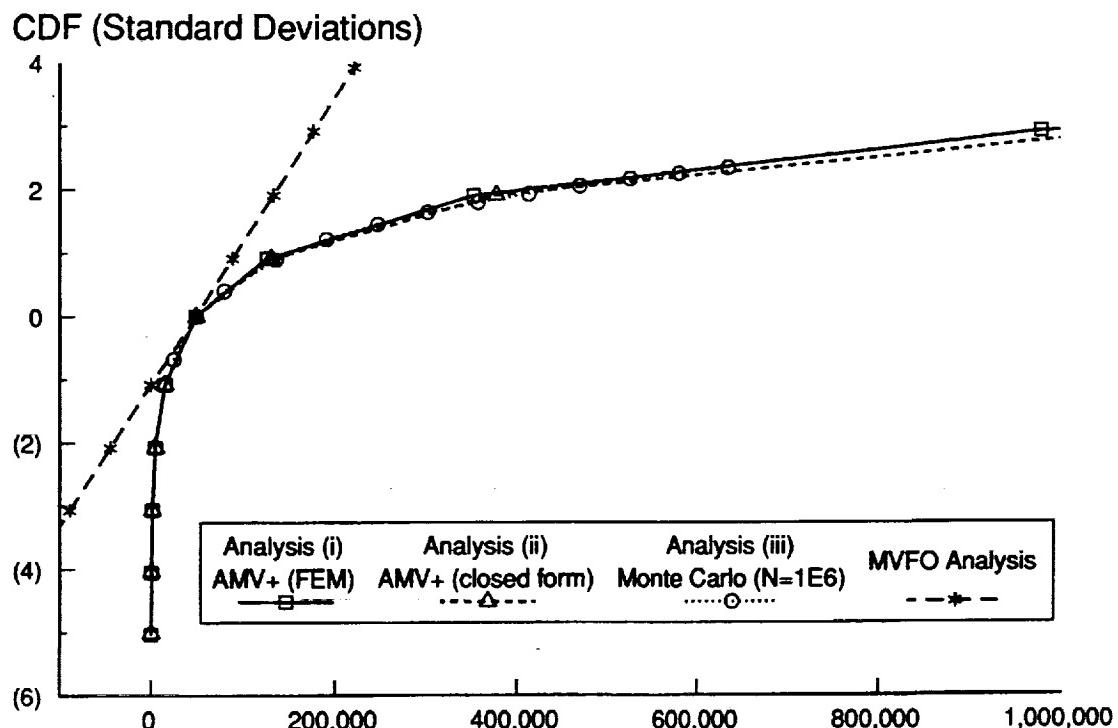


Figure 2.9 - CDF of the Number of Cycles to Failure for the Three Point Bend Problem

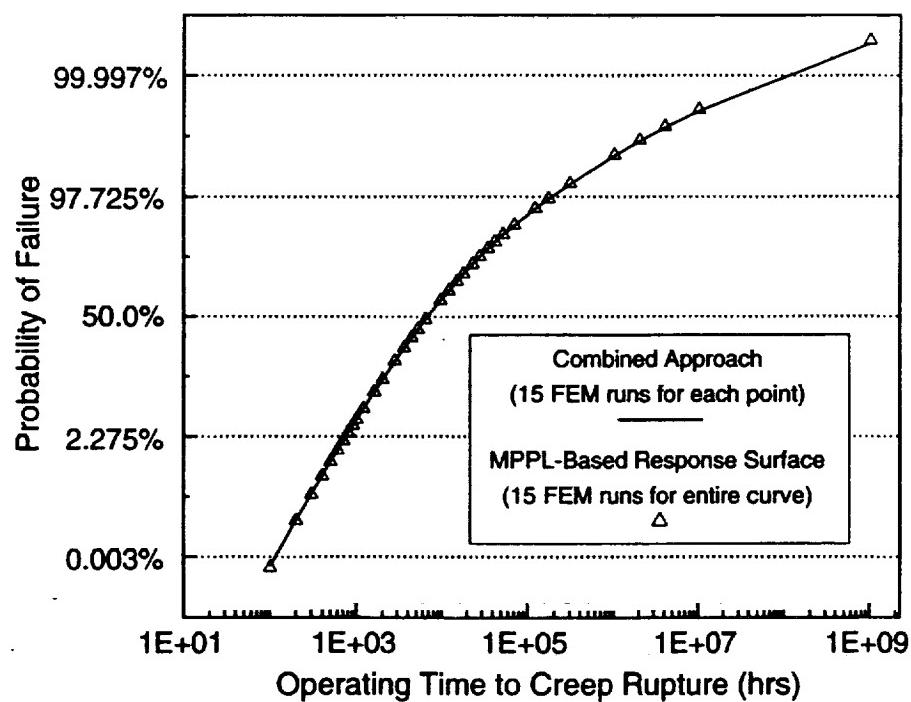
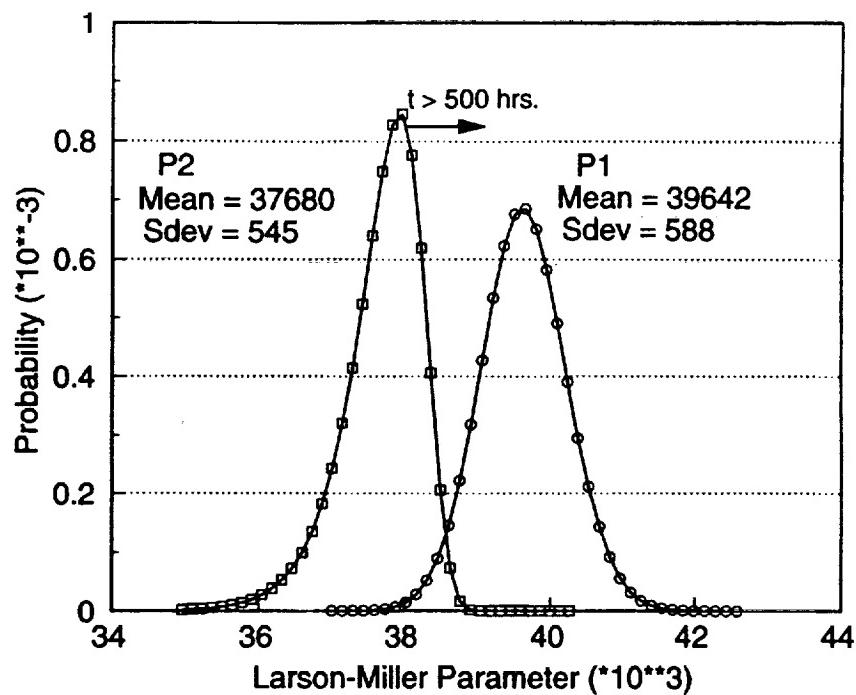


Figure 2.10 - CDF of Service Life of a Turbine Blade Subject to Creep Rupture

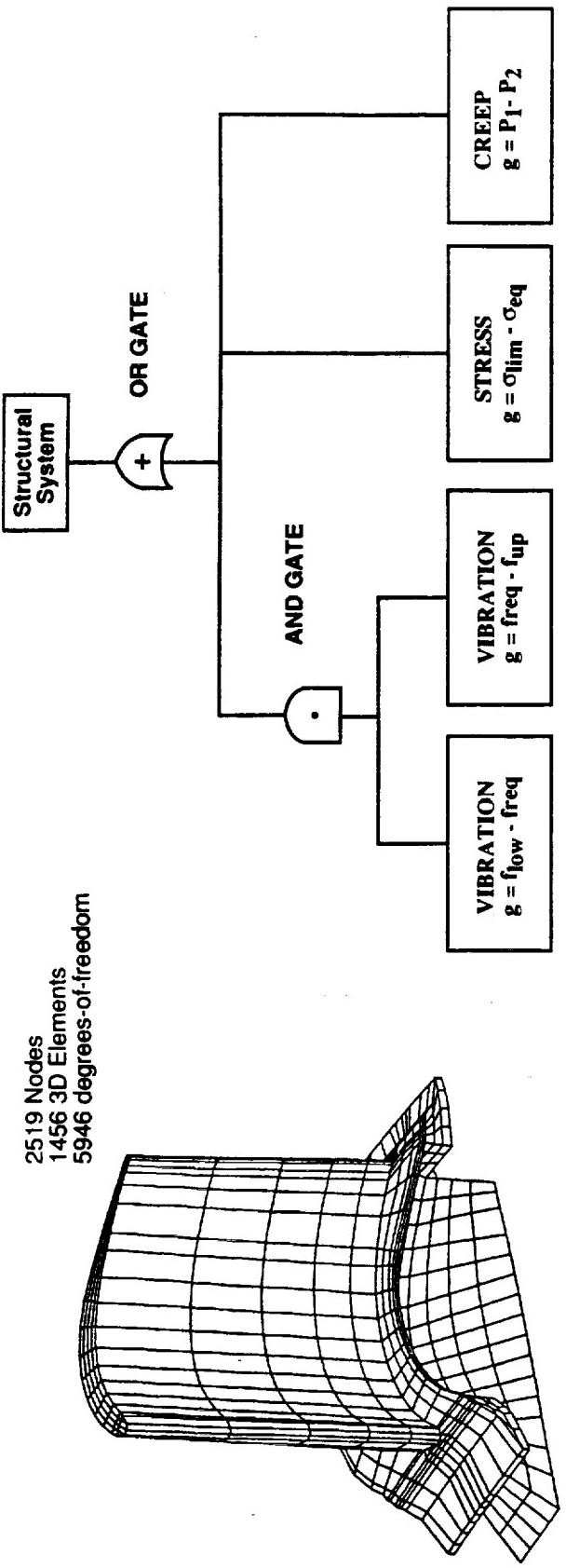
In 1992-1993, NESSUS, version 6.0, was enhanced to include system reliability capabilities. System reliability is defined here in terms of failure due to multiple locations, multiple failure modes or multiple components. One approach implemented was the probabilistic fault tree analysis approach (PFTA). PFTA provides a logical means to organize and define the definition of the system failure. The failure tree is defined in terms of AND and OR gates. The inputs into the gates are the structural models defined using the NESSUS capabilities described above or the output from other gates. The PFTA procedure is described in References 2.12-2.13.

A new probabilistic method, the Adaptive Importance Sampling (AIS) method was developed and implemented to compute the system reliability efficiently. AIS is described in References 2.15-2.16.

Several application problems were analyzed using the system approach. One detailed example of the system reliability of a turbine blade subject to stress, vibration and creep failure modes is presented in Reference 2.21. Figure 2.11 shows the finite element model, the fault tree and the probabilistic results.

Probabilistic structural analysis can be very computationally time consuming even with efficient probabilistic methods. Therefore, an approach was developed and described in Reference 2.22 to use a combination of coarse (global) and fine (local) finite element models during a probabilistic analysis. The global/local approach uses the Most Probable Point from the global model as a starting point for the local analysis. The result is that if the global and local models are highly correlated, the global model can be used for most of the calculations and the local modeled used for selective updating. An example is shown in Figure 2.12 in which the combined global/local model predicts the same probabilistic results as the local model with significantly less computer time. This methodology has not been automated in NESSUS but the concepts and theoretical basis have been demonstrated.

Structural analysis often involves life prediction in terms of cycles to failure. An efficient, accurate fracture mechanics methodology was developed and used to analyze the distribution of remaining life of a turbine blade attachment as shown in References 2.23 and 2.24. The crack size distribution as a function of time was computed and the effect of in-service inspection simulated. This approach can be used to determine the probability of failure before and after inspection, the effectiveness of various inspection techniques, and an optimized inspection schedule. Figure 2.13 shows the structural model, the CDF of fatigue life, and the crack size distributions before and after inspection.

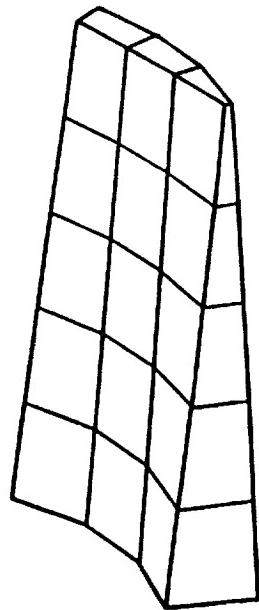


## System Reliability Results

Analysis Type	Bottom Events	System Samples	Probability of Failure	Total CPU
Monte Carlo	Approximate	1721/100000	0.01721	2.5 hrs
AIS	Approximate	70/83	0.01711	2.5 hrs
AIS	Finite Element	90/145	0.02387	30 hrs

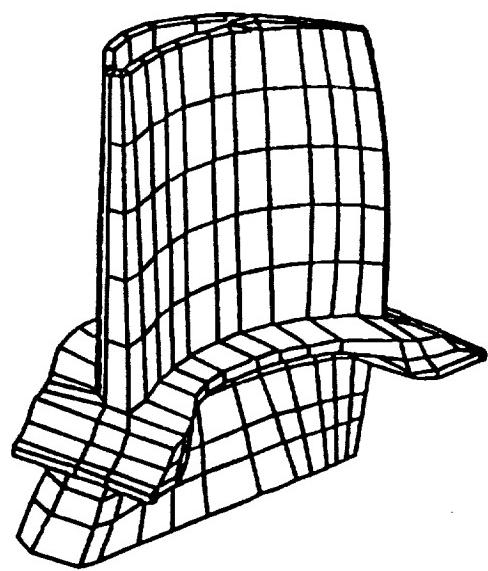
Figure 2.11 - Model of Structural System Failure and System Reliability Results

15 elements  
48 nodes



Coarse Mesh

359 elements  
811 nodes



Fine Mesh

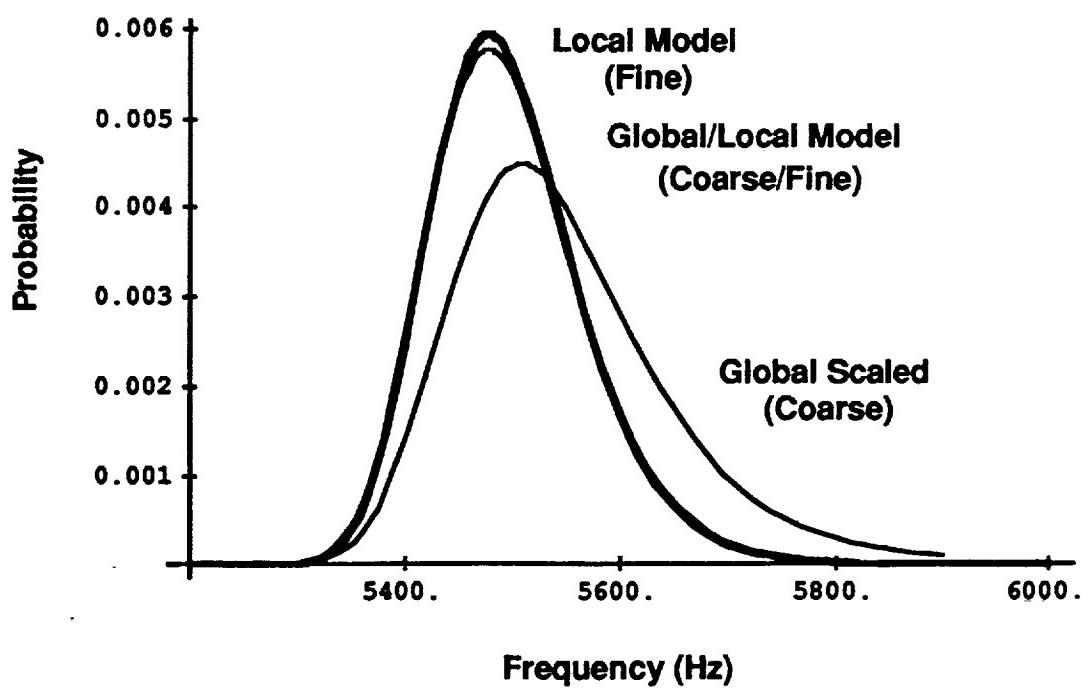


Figure 2.12 - Global/Local Approach and Results

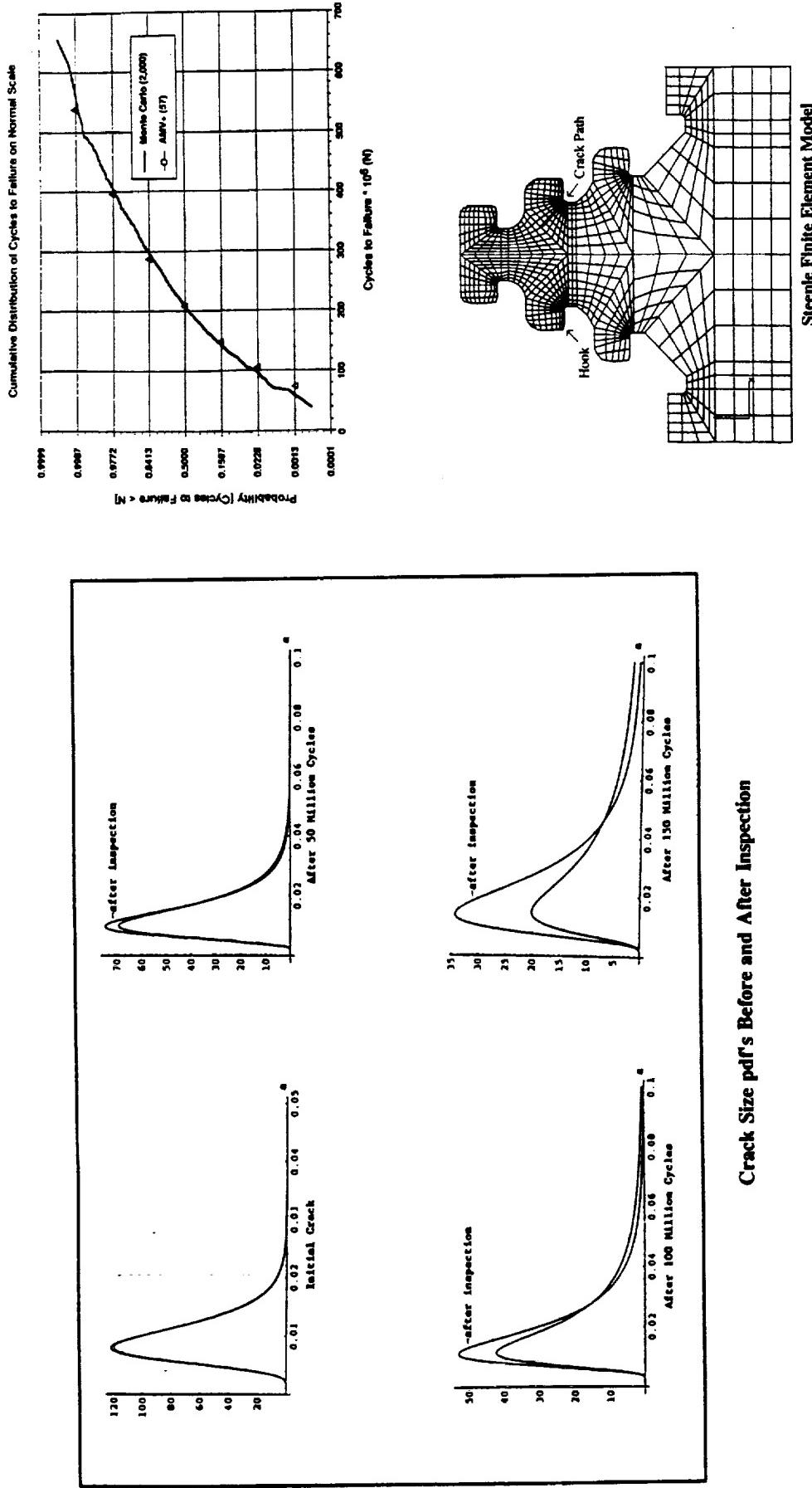


Figure 2.13 - Probabilistic Fracture Mechanics Analysis

## References

- 2.1 Dias, J., Nagtegaal, J., and Nakazawa, S., "Iterative Perturbation Algorithms in Probabilistic Finite Element Analysis," Computational Mechanics of Reliability Analysis, W. K. Liu and T. Belytschko (eds.), Elmepress International, Ch. 9, 1989.
- 2.2 Cruse, T. A., and Huang, Q., "Use of the Boundary Element Method for Probabilistic Structural Analysis," 1992 Conference on Advanced Earth-to-Orbit Propulsion Technology, Huntsville, AL, May 19-21, 1992.
- 2.3 Newell, J. F. Newell, Rajagopal, K. R., and Ho, H., "Probabilistic Structural Analysis of Space Propulsion System Turbine Blades," *Proceedings, 31st Structures, Structural Dynamics and Materials Conference*, Long Beach, CA, paper 90-1101-CP, April 2-4, 1990.
- 2.4 DebChaudhury, A., Rajagopal, K. R., Ho, H., and Newell, J. F., "A Probabilistic Approach to the Dynamic Analysis of Ducts Subjected to Multibase Harmonic and Random Excitation," *Proceedings, 31st Structures, Structural Dynamics and Materials Conference*, Long Beach, CA, paper 90-1100-CP, April 2-4, 1990.
- 2.5 Rajagopal, K., Keremes, J., Ho, H., and Orient, G., "A Probabilistic Approach to the Evaluation of Fatigue Damage in Space Propulsion System Injector Element," *Proceedings, 33rd Structures, Structural Dynamics and Materials Conference*, Dallas, TX, April 13-15, 1992.
- 2.6 Wu, Y.-T., and Burnside, O., "Validation of the NESSUS Probabilistic Finite Element Analysis Computer Program," *Proceedings, IAAA/ASME/ASCE/AHS/ASC 29th Structures, Structural Dynamics, and Materials Conference*, Williamsburg, VA, Part 3, pp. 1267-1274, Paper No. 88-2372, April 18-20, 1988.
- 2.7 Millwater, H., Wu, Y.-T., and Fossum, A., "Probabilistic Analysis of a Materially Nonlinear Structure," *Proceedings AIAA/ASME/AHS/ASC 31st Structures, Structural Dynamics, and Materials Conference*, Long Beach, CA, Paper No. 90-1099-CP, April 2-4, 1990.
- 2.8 Wu, Y.-T., Millwater, H. R., Cruse, T. A., "An Advanced Probabilistic Structural Analysis Method for Implicit Performance Functions," *AIAA/ASME/ASCE/ASC 30th Structures, Structural Dynamics, and Materials Conference*, Mobile, AL, Paper No. AIAA 89-1371, April 3-5, 1989; *AIAA Journal*, Vol. 28, No. 9, 1840-1845, Sept 1990.
- 2.9 Wu, Y.-T., and Burnside, O. H., "Efficient Probabilistic Structural Analysis using an Advanced Mean Value Method," *Proceedings, ASCE Specialty Conference on Probabilistic Mechanics and Structural and Geotechnical Safety*, pp. 492-495, 1988.
- 2.10 Wu, Y.-T., Burnside, O. H., and Cruse, T. A., "Probabilistic Methods for Structural Response Analysis," Computational Mechanics of Reliability Analysis, W. K. Liu and T. Belytschko (eds.), Elmepress International, Ch. 7, 1989.
- 2.11 Southwest Research Institute, NESSUS 6.0 Release Notes, June 1992. (Describes system reliability analysis using fault tree method and Latin Hypercube sampling of PFEM model).

- 2.12 Tornq, T., Wu, Y.-T., and Millwater, H., "Structural System Reliability Calculation using a Probabilistic Fault Tree Analysis", *Proceedings 33rd Structures, Structural Dynamics, and Materials Conference*, Dallas, TX, April 13-15, 1992.
- 2.13 Millwater, H. R., and Wu, Y.-T., "Structural Reliability Assessment Capability in NESSUS," Presented at the 1992 JPC AIAA/SAAE/ASME/ASEE 28th Joint Propulsion Conference, Nashville, TN, July 6-8, 1992.
- 2.14 Wu, Y.-T., Tornq, T. Y., Burnside, O. H., and Rheinfurth, M. H., "Probabilistic Rotordynamics Analysis Using an Adaptive Importance Sampling Method," Presented at the 1992 ASCE Speciality Conference on Probabilistic Mechanics and Structural Geotechnical Reliability, Denver, CO, July 1992.
- 2.15 Wu, Y.-T., "An Adaptive Importance Sampling Methods for Structural System Reliability Analysis," Presented at the 1992 ASME Winter Annual Meeting, Anaheim, CA, November 8-11, 1992.
- 2.16 Wu, Y.-T., "Computational Methods for Efficient Structural Reliability and Reliability Sensitivity Analysis," Presented at the 34th SDM Structures, Structural Dynamics, and Materials Conference, La Jolla, CA, April 19-21, 1993; *AIAA Journal*, Vol. 32, No 8, pp. 1717-1723.
- 2.17 Southwest Research Institute, NESSUS User's Manual, 1995.
- 2.18 Thacker, B. H., Millwater, H. R., and Harren, S. V., "Computational Methods for Structural Load and Resistance Modeling," Presented at the 32nd Structures, Structural Dynamics, and Materials Conference, Baltimore, Maryland, Paper No. AIAA-91-0918, April 7-10, 1991.
- 2.19 Millwater, H. R., Harren, S. V., and Thacker, B. H., "Probabilistic Analysis of Structures Involving Random Stress-Strain Behavior," Presented at the 32nd Structures, Structural Dynamics and Materials Conference, Baltimore, Maryland, Paper No. AIAA-91-0919, April 7-10, 1991.
- 2.20 Thacker, B. H., and Wu, Y.-T., "A New Response Surface Approach for Structural Reliability Analysis," Presented at the 33rd Structures, Structural Dynamics and Materials Conference, Dallas, Texas, April 13-15, 1992.
- 2.21 Millwater, H. R., and Wu, Y.-T., "Computational Simulation of Structural Reliability Analysis of a Turbine Blade, Presented at the 1993 IGTI 38th International Gas Turbine Conference, Cincinnati, OH, May 24-27, 1993.
- 2.22 Millwater, H. R., and Wu, Y.-T., "Global/Local Methods for Probabilistic Structural Analysis," Presented at the 34th SDM Structures, Structural Dynamics, and Materials Conference, La Jolla, CA, April 19-21, 1993.
- 2.23 Millwater, H. R. Millwater, Wu, Y.-T., and Cardinal, J. W., "Probabilistic Structural Analysis of Fatigue and Fracture," Presented at the 35th SDM Structures, Structural Dynamics, and Materials Conference, Hilton Head, SC, April 18-20, 1994.

- 2.24 Millwater, H. R. Millwater, Wu, Y.-T., Cardinal, J. W., and Chell, G. G., "Application of Advanced Probabilistic Fracture Mechanics to Life Evaluation of Turbine Rotor Blade Attachments," Presented at the 1995 IGTI 40th International Gas Turbine Conference, Houston, June 5-8, 1995.

### 3. PROBABILISTIC ANALYSIS METHODOLOGY

#### 3.1 Introduction

This section summarizes the following probabilistic methods in the NESSUS code.

- First-order reliability method
- Second-order reliability method
- Advanced first-order method using three-parameter normal
- Convolution methods
- Standard Monte Carlo method
- Radius-based importance sampling method
- Adaptive importance sampling methods
- Advance mean-based method
- System reliability analysis method

Most of the methods are approximate in nature. Based on the Fast Probability Integration (FPI) concept, these methods usually provide very efficient analysis relative to the standard Monte Carlo method which provides asymptotically exact solution as the number of random sampling approaches infinity.

The selection of methods must consider required computational time, which strongly depends on the complexities of the performance functions (e.g., finite element models versus closed form equations), and solution accuracy, which generally depends on the nonlinearity and smoothness of the performance functions.

#### 3.2 Basic Definitions and Fast Probability Integration (FPI) Concept

##### 3.2.1 Response, Performance, and Limit-state Function

The FPI concept was originated from structural reliability analysis where failure conditions must be pre-defined using limit states [References 3.1-3.4]. To expand the concept to CDF (cumulative distribution function) analysis, this document adopts the following definitions and notations that distinguish response functions from limit state functions.

A Z-function is a response function or a performance function such as stress, displacement, natural frequency, fatigue life,.. etc.

$$Z(\mathbf{X}) = Z(X_1, X_2, X_3, \dots, X_n) \quad (3.1)$$

where  $X_i$  ( $i = 1, n$ ) are the random variables.

A g-function is a limit state (also called performance function in the literature) defined as:

$$g = Z(\mathbf{X}) - z_o = 0 \quad (3.2)$$

where  $z_o$  is a particular value of  $Z$ . The  $g$ -function is defined such that  $g(\mathbf{X}) = 0$  is a boundary that divides the random variable space into two regions: failure [ $g \leq 0$ ] and safe [ $g > 0$ ]. Because the CDF of  $Z$  at  $z_o$  equals the probability that [ $g \leq 0$ ], the CDF can be computed by varying  $z_o$  and computing the point probability.

A component reliability problem has only one  $g$ -function whereas a system reliability problem involves multiple  $g$ -functions.

### 3.2.2 Probability Integration

Given a  $g$ -function and given the joint probability density function, PDF,  $f_{\mathbf{X}}(\mathbf{x})$ , the probability of failure is the probability in the failure domain  $\Omega$  and is given by:

$$p_f = \int_{\Omega} \dots \int f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (3.3)$$

This integral can be computed using a straightforward standard Monte Carlo procedure. However, when the  $g$ -function is complicated, requiring an intensive numerical calculation for each sample of  $\mathbf{X}$ , and  $p_f$  is small, this random sampling procedure becomes impractical for engineering analysis and design. For practical purposes, efficient and approximate analysis tools are needed.

### 3.2.3 Most Probable Point Concept

Several methods in NESSUS are based on the Most Probable Point (MPP) concept. In the structural reliability literature, the MPP is also known as the Design Point, which is defined in a coordinate system of an independent, standardized normal vector  $\mathbf{u}$ . In the  $\mathbf{u}$ -space, the joint probability density function (PDF) is rotationally symmetric around the origin and decays exponentially with the square of the distance from the origin. For a two-variable case, the joint PDF has a bell-shape surface.

By transforming  $g(\mathbf{X})$  to  $g(\mathbf{u})$  using a distribution transformation, the MPP ( $\mathbf{u}^*$ ) is a point that defines the minimum distance,  $\beta$ , from the origin ( $\mathbf{u} = 0$ ) to the limit-state surface  $g(\mathbf{u}) = 0$ . This minimum-distance point is a most-probable-point on  $g(\mathbf{u}) = 0$  (in the  $\mathbf{u}$ -space) because the joint PDF at a point  $(u_1, u_2, \dots, u_n)$  in the  $\mathbf{u}$ -space is proportional to  $\exp\{-0.5(u_1^2 + u_2^2 + \dots + u_n^2)\}$  where the sum of squares defines the distance. Therefore the density is a maximum when the distance is a minimum. The MPP concept is illustrated in Figure 3.1.

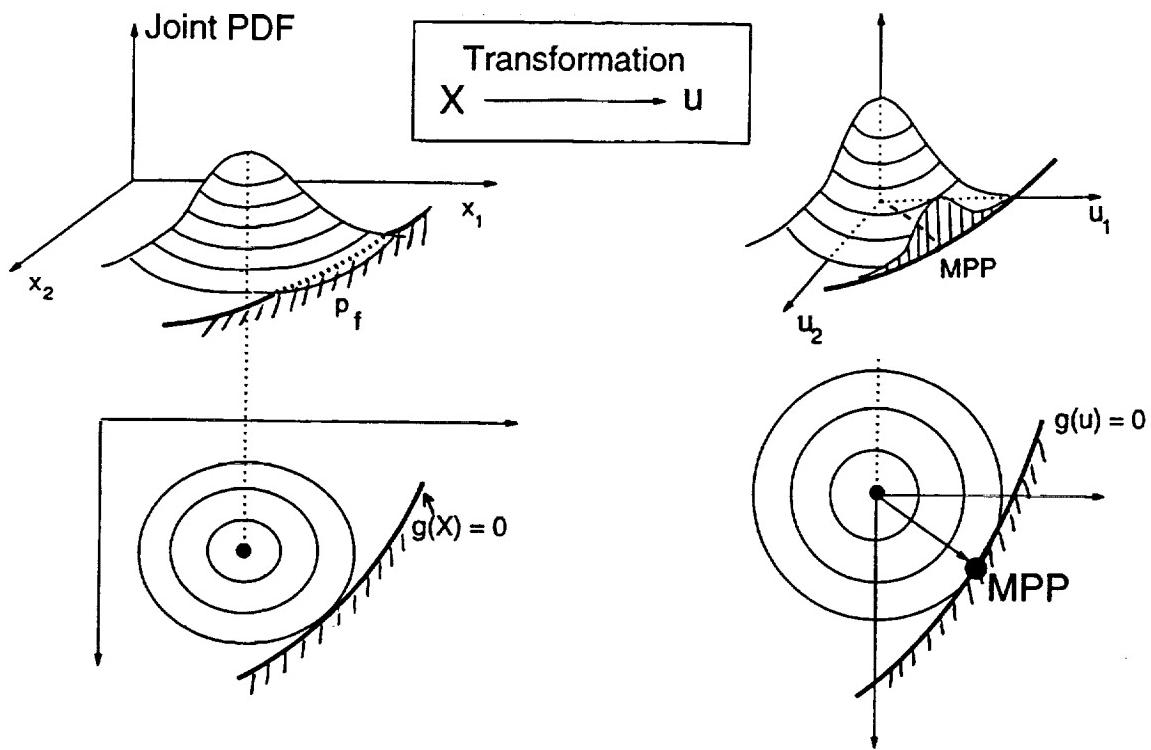


Figure 3.1 - Most Probable Point (MPP)

Because of its unique probability properties, the MPP is a key point for fast probability analysis. Once the MPP is identified, it can be used as a basis to develop approximate polynomial  $g$ -functions. The approximate  $g$ -functions can be analyzed more easily using several methods including the first and second-order reliability methods and the convolution methods. The MPP can also be used as a basis for the adaptive sampling methods described in Section 3.8.

### 3.2.4 Distribution Transformation

Non-normal dependent variables  $\mathbf{X}$  can be transformed to standardized normal variables  $\mathbf{u}$  using the Rosenblatt transformation [References 3.3-3.6]:

$$u_1 = \Phi^{-1}[F_1(x_1)]$$

$$u_2 = \Phi^{-1}[F_2(x_2 | x_1)] \quad (3.4)$$

$$u_n = \Phi^{-1}[F_n(x_n | x_1, x_2, \dots, x_{n-1})]$$

where  $F_i(x_i)$  is the CDF of  $X_i$ ,  $F_n(x_n | \dots)$  is the conditional CDF, and  $\Phi^{-1}(\cdot)$  is the inverse CDF of a standardized normal random variable.

When the variables are mutually independent, this transformation reduces to:

$$\mathbf{u} = \Phi^{-1}[\mathbf{F}_X(\mathbf{x})] \quad (3.5)$$

The inverse transformation is:

$$\mathbf{x} = \mathbf{F}_X^{-1}[\Phi(\mathbf{u})] \quad (3.6)$$

Using the above transformation, the entire  $g(\mathbf{X})$ -function can be transformed to  $g(\mathbf{u})$  and allow the probabilistic analysis to be performed in the  $\mathbf{u}$ -space. Numerically, however, the  $\mathbf{X}$ -to- $\mathbf{u}$  or  $\mathbf{u}$ -to- $\mathbf{X}$  transformations are needed only at points required to find the MPP, construct polynomials, and perform importance sampling.

The advantage for transforming to the  $\mathbf{u}$ -space is that probabilistic analysis becomes mathematically more tractable. The drawback is that the involved transformation may significantly distort the  $g$ -function such that an originally flat surface becomes highly curved.

For engineering applications, the Rosenblatt transformation for dependent random variables may be impractical because the available data is often insufficient to establish the joint and the conditional probability distributions. A more realistic model transforms each correlated, non-normal random variable into a normal variable and generates a new set of correlation coefficients for the transformed normals [References 3.7-3.9]. The generated normal variables are then assumed to have a joint normal density function (which is generally not true) and the correlation coefficients are used to generate a set of independent normal random variables. The inputs required for the second option include only the marginal distributions and the correlation coefficients. This option gives exact solutions for correlated normal random variables.

Consider two random variables  $X_i$  and  $X_j$  with correlation coefficient  $R$ . The correlation coefficient of the transformed normal variables  $U_i$  and  $U_j$ , denoted as  $r$ , can be found by solving the following equation:

$$R = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \frac{x_i - \mu_i}{\sigma_i} \right) \left( \frac{x_j - \mu_j}{\sigma_j} \right) \phi_{ij} du_i du_j \quad (3.7)$$

where

$$\phi_{ij} = \frac{1}{2\pi\sqrt{1-r^2}} \exp\left(-\frac{u_i^2 - 2ru_i u_j + u_j^2}{2(1-r^2)}\right) \quad (3.8)$$

In general, the calculation of  $r$  requires iteratively solving the above equation. A convenient approach relating  $R$  and  $r$  was proposed in Reference 3.10 and implemented in NESSUS.

### 3.2.5 Most Probable Point (MPP) Search Procedure

The MPP is the key approximation point for the FPI analysis. Therefore, the identification of the MPP is an important task. In general, the identification of the MPP can be formulated as a standard optimization problem (i.e., find the minimum distance point subject to  $g=0$ ) and can be solved by many optimization methods. In the NESSUS code, there are two procedures for finding the MPP. One is implemented specifically for the AMV methods (described in Section 3.9) and the other is summarized in the following steps:

- (a) Assume an MPP  $\mathbf{x}^*$ .
- (b) Compute equivalent normal distributions for the non-normal random variables at  $\mathbf{x}^*$ .
- (c) Construct a linear  $g$ -function using  $\mathbf{x}^*$  as the expansion point.
- (d) Based on the equivalent normal distributions, compute the minimum distance and the updated MPP in the  $u$ -space.
- (e) Repeat steps (b) to (d) until the MPP converges.

In (c), the equivalent normal distribution is computed by the Rackwitz-Fiessler algorithm [References 3.1, 3.3, 3.10] which matches the CDF's and the PDF's for the original and the equivalent normal distribution at the MPP point,

$$F_x(x^*) = \Phi(u^*) \quad (3.9)$$

and

$$f_x(x^*) = \frac{\phi(u^*)}{\sigma_N} \quad (3.10)$$

where

$$u^* = \frac{x^* - \mu_N}{\sigma_N} \quad (3.11)$$

The parameters  $\mu_N$  and  $\sigma_N$  are the mean and the standard deviation of the approximate normal and  $\phi(\cdot)$  is the standard normal PDF.

From the above three equations, the approximate normal parameters can be derived as:

$$\sigma_N = \frac{\phi[\Phi^{-1}(F_x(x^*))]}{f_x(x^*)} \quad (3.12)$$

$$\mu_N = x^* - \Phi^{-1}[F_x(x^*)]\sigma_N \quad (3.13)$$

According to the principle of normal tail approximation [Reference 3.11], the MPP obtained by the approximate normal is identical to the one obtained using the inverse transformation. In the equivalent normal approach, which is the approach used in the NESSUS code, the inverse transformation is not needed.

The above iteration algorithm has been found to be quite efficient and robust. Many other algorithms have been proposed in the literature [References 3.12-3.14]. However, just like using any optimization methods, there is no guarantee that the procedure will always converge. Furthermore, the possibility exists that there are multiple (local and global) MPP's. The NESSUS code assumes only one MPP. Some multiple MPP problems may be identified and solved by using the AMV procedure described in Section 3.9. In general, experience is required in selecting methods. For non-convergent problems, the AMV methods should be tried followed by the importance sampling method (described in Section 3.7), and, as the last resort, the standard Monte Carlo method.

### 3.2.6 Probability Sensitivity Analysis

In deterministic analysis, sensitivity is defined as  $\partial Z / \partial X_i$ , which measures the change in the performance due to the change in a design parameter. In probabilistic analysis the sensitivity measure is  $\partial p / \partial \theta_i$ , which measures the change in the probability relative to the change in a distribution parameter (e.g., mean and standard deviation).

Another useful probability sensitivity analysis is the determination of the relative importance of the random variables. This can be done by performing several probabilistic analyses in which one of the random variables is treated as a deterministic variable (i.e., by

reducing the standard deviation to zero) for each analysis. Based on the resulting probability changes, the relative importance of the random variables can be determined. Repeated analyses, however, may be very time consuming for large number of random variables.

A more efficient way of evaluating the relative importance of the random variables is based on the location of the MPP. At the MPP,  $\mathbf{u}^* = (u_1^*, u_2^*, \dots u_n^*)$ , the first-order probability estimate is  $\Phi(-\beta)$  where

$$\beta^2 = u_1^{*2} + u_2^{*2} + \dots u_n^{*2} \quad (3.14)$$

The unit normal vector at the MPP of the  $g=0$  surface is defined as:

$$\alpha = -\frac{\nabla g}{|\nabla g|} \quad (3.15)$$

The  $\alpha$  vector is positive towards the direction of decreasing  $g$  (i.e., to the failure region). The sensitivity factors are the projections of the  $\alpha$  vector to the  $u$ -axes. Thus, they are the directional cosines of the  $\alpha$  vector as shown in Figure 3.2, and can be written as:

$$\alpha_i = \frac{u_i^*}{\beta} \quad (3.16)$$

The directional cosines satisfy the following rule:

$$\alpha_1^2 + \alpha_2^2 + \dots \alpha_n^2 = 1 \quad (3.17)$$

which implies that each  $\alpha_i^2$  is a measure of the contribution to the probability (since the probability is related to  $\beta$ ); higher  $\alpha$  (in magnitude) indicates higher contribution. Thus, the sensitivity factors provide a first-order information on the importance of the individual random variable.

It can be shown that in the  $u$ -space,

$$|\alpha_i| \propto \left( \frac{\partial g}{\partial u_i} \right)_u \quad (3.18)$$

and in the  $X$ -space,

$$|\alpha_i| \propto \left( \frac{\partial g}{\partial X_i} \right)_x \sigma_i \quad (3.19)$$

where  $\sigma_i$  is the normal (or approximate normal for a non-normal distribution) standard deviation. It can be concluded that the sensitivity factors are functions of *both* the deterministic sensitivity and the uncertainty (characterized by the standard deviation).

In general, the sensitivity factors depend on the g-function as well as the input probability distributions. In a CDF analysis, the sensitivity factors will usually be different for different response or probability levels. This is because the performance sensitivity or the approximate standard deviation may be different for different levels.

Because the above probabilistic sensitivity analysis is based on the first-order reliability method,  $\alpha$  is a good probability sensitivity measure only if  $\Phi(-\beta)$  is a good approximation to the true probability (see Section 3.3 for more information).

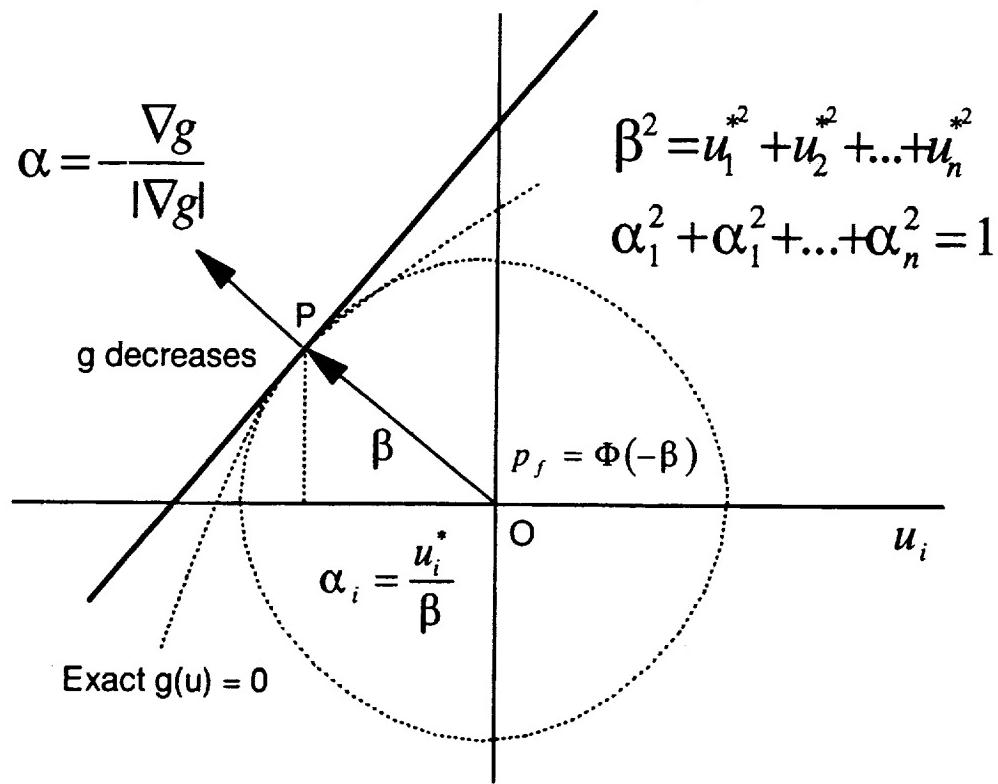


Figure 3.2 - The Definition of Sensitivity Factors

Based on the MPP, other sensitivity measures with respect to a distribution parameter (mean or standard deviation) or a limit-state function parameter can be computed based on the sensitivity factors and the distribution transformation [Reference 3.4]. However, the computation has not yet been implemented. Section 3.8 describes a sampling-based method for probabilistic sensitivity analysis.

### 3.3 First and Second-order Reliability Methods (FORM and SORM)

The FORM probability solution is based on the linearization of the g-function at the most probable point (MPP) in the u-space. The first-order polynomial,  $g_1(\mathbf{u})$ , is:

$$g_1(\mathbf{u}) = a_o + \sum_{i=1}^n a_i(u_i - u_i^*) \quad (3.20)$$

Given  $g_1(\mathbf{u})$ , the probability of failure is a function of the minimum distance to the plane defined by  $g_1$  in the u-space [References 3.3, 3.4]:

$$p_f = \Phi(-\beta) \quad (3.21)$$

where  $\beta$  is computed from

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{a_o + \sum_{i=1}^n a_i \mu_i}{\sqrt{\sum_{i=1}^n a_i^2 \sigma_i^2}} \quad (3.22)$$

which is allowed to take negative values. A negative  $\beta$  means the origin is in the failure region (i.e., for the  $p_f > 0.5$  case) and the  $\alpha$  vector is positive from the MPP to the origin. Figure 3.3 summarizes the FORM results.

The SORM analysis requires a second-order polynomial,  $g_2(\mathbf{u})$ ,

$$g_2(\mathbf{u}) = a_o + \sum_{i=1}^n a_i(u_i - u_i^*) + \sum_{i=1}^n b_i(u_i - u_i^*)^2 + \sum_{i=1}^n \sum_{j=1}^{i-1} c_{ij}(u_i - u_i^*)(u_j - u_j^*) \quad (3.23)$$

The definition of  $g_2(\mathbf{u})$  requires  $n(n-1)/2$  second order derivatives. NESSUS computes the second-order coefficients by numerical differentiation. Given  $g_2(\mathbf{u})$ , approximate or exact solutions are available [References 3.15-3.16]. A special case of  $g_2(\mathbf{u})$  is a parabola which has an exact integral solution for  $p_f$  [Reference 3.17]. Breitung derived the following asymptotic formula for large  $\beta$  [Reference 3.17]:

$$p_f \approx \Phi(-\beta) \prod_{i=1}^{n-1} (1 - \beta \kappa_i)^{-1/2} \quad (3.24)$$

where  $\kappa_i$  are the principal curvatures at the most probable point.

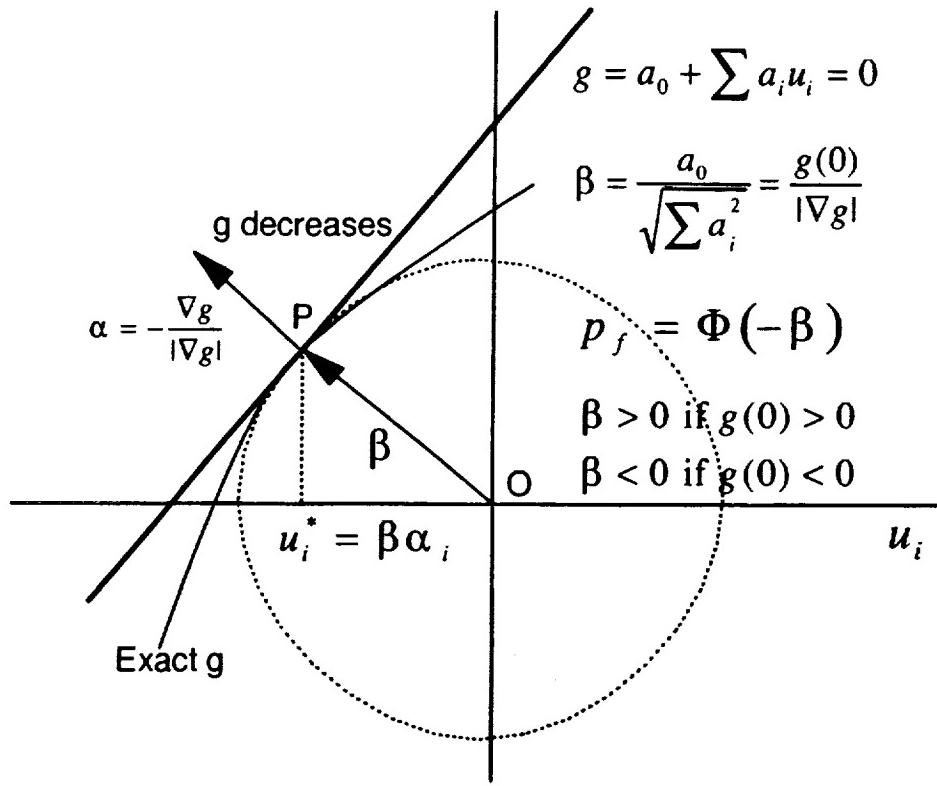


Figure 3.3 - First Order Reliability Method

The first and second-order methods, as well as other MPP-based methods (including the advanced first-order, the convolution method, and the AMV methods), attempt to approximate the original  $g$ -functions by simple functions. It has been reported that these MPP-based methods perform well for a wide range of applications. However, it should be cautioned that some nonlinear functions (e.g., a sine wave function) cannot be approximated well by low-order polynomials. In practice, when a performance function is implicitly defined and is highly nonlinear, it is difficult to assess the error introduced by the polynomial approximation. Therefore, for any new type of  $g$ -functions, the approximate solutions should be checked using other more accurate methods.

### 3.4 Advanced First-order Reliability Method

In this method, the  $g$ -function is first approximated by a quadratic function at the MPP,  $x^*$ . Then the quadratic function is transformed to a linear function by using intermediate variables. Afterwards, a three-parameter normal CDF is constructed for each non-normal intermediate random variable in the transformed linear function. Finally, the probability is approximated by integration using the three-parameter normals [Reference 3.20].

This method is called the advanced first-order method because the linear g-function is obtained by transformation from a quadratic function, whereas in the FORM approach, the g-function is linearized by truncation (i.e., the g-function is replaced by a hyper-plane in the u-space). This method is primarily a X-based method, and has been used in conjunction with the AMV methods. Experience suggests that this method is more accurate than FORM or SORM for the type of g(X)-functions that are nearly linear or can be linearized (by using intermediate variables). In general, like FORM and SORM, it is difficult to assess the error introduced by the polynomial approximation and transformation. Therefore, for any new type of g-functions, the solutions should be checked using other more accurate methods.

For a linear g-function with all  $X_i$  begin normally distributed, then  $p_f = \Phi(-\beta)$ , where

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{a_o + \sum_{i=1}^n a_i \mu_i}{\sqrt{\sum_{i=1}^n a_i^2 \sigma_i^2}} \quad (3.25)$$

For all non-normal variables, we wish to establish high-quality approximate normals. The approach is to fit the original CDF,  $F_x(x)$ , by an approximating function  $F'(x)$  which is a three-parameter normal,

$$F_x(x) \approx F'(x) = \gamma \Phi(u) \quad (3.26)$$

where  $u = (x - \mu_N)/\sigma_N$  and  $\gamma$ ,  $\mu_N$ , and  $\sigma_N$  are the three parameters to be determined. The three-parameter normal distribution is a normal distribution multiplied by a scale factor. The scale factor provides more flexibility in fitting a non-normal distribution and can be taken out of the integrand in the probability of failure integral.

The three parameters are computed by solving an optimization problem in which the integral of the square of the difference between  $F'(x)$  and  $F(x)$ , both multiplied by a suitable weight function,  $W(x)$ , is minimized. The role of the weight function is to determine a better fit. For each non-normal variable,  $W(x)$  is determined based on the location of the MPP, and  $W(x)$  is different for each random variable.

After obtaining all the parameters,  $p_f$  is estimated as

$$p_f = \Phi(-\beta') \prod_{i=1}^n \gamma_i \quad (3.27)$$

where  $\beta'$  is computed using  $\mu_N$ , and  $\sigma_N$ .

For a nonlinear g-function, the Z-function is first approximated by a second-order polynomial using Taylor's series expansion with the MPP,  $x^*$ , as the expanding point:

$$Z(\mathbf{X}) = \sum_{i=1}^n a_i (X_i - x_i^*) + b_i (X_i - x_i^*)^2 \quad (3.28)$$

where  $a_i$  and  $b_i$  are coefficients that can be computed by numerical differentiation. After rearranging terms,

$$Z(\mathbf{X}) = \sum_{i=1}^n -\frac{a_i^2}{4b_i} + \sum_{i=1}^n b_i \left[ X_i - \left( x_i^* - \frac{a_i}{2b_i} \right) \right]^2 \quad (3.29)$$

which can be transformed to a linear form:

$$Z(\mathbf{Y}) = c_o + \sum_{i=1}^n c_i Y_i \quad (3.30)$$

where the intermediate variables are defined as:

$$Y_i = X_i - \left( x_i^* - \frac{a_i}{2b_i} \right)^2 \quad (3.31)$$

which defines the transformation from  $X_i$  to  $Y_i$ , where  $Y_i$  is a function of  $X_i$  only. Thus,  $Z(\mathbf{Y})=0$  is linear; and  $Y_i$ 's are independent if  $X_i$ 's are independent. Given the CDF of  $X_i$ , the CDF of  $Y_i$  can be easily computed by probability transformation rules.

The above second-order polynomial does not involve mixed, or cross-product terms. If a full quadratic function is obtained in the  $u$ -space, a coordinate transformation by a rotation can be used to generate a new set of independent  $u$ -variables such that the mixed terms disappear in the transformed  $g$ -function. In such a case, the above method would produce an approximate SORM solution. However, the  $u$ -space transformation option has not been implemented in the current version of NESSUS, instead, it has been implemented in the convolution method (see Section 3.5).

The above concept allows additional transformations such as log-transformations or using other intermediate random variables to make the  $g$ -function more linear and therefore can potentially take into account higher than second-order effects. The objective for the transformations is to make the function more close to linear and let the three-parameter normal handle the remaining probability integration analysis. Further discussions can be found in References 3.18-3.20.

### 3.5 Fast Convolution Method

The fast convolution method in NESSUS applies the convolution theorem and the fast Fourier transform (FFT) technique to efficiently perform probabilistic analysis of a linearized performance function. Linearized functions can be constructed in the X-space or in the u-space. In the X-space a partial second-order approximation function is used to approximate the original performance function around the MPP. In the u-space a full second-order approximation function is used. In both approaches, the second-order functions are linearized using intermediate variable procedures (see Section 3.4).

The convolution theorem provides exact CDF results (in one-dimensional integral form) for a sum of independent random variables. The convolution procedure implemented in NESSUS consists of the following steps: (1) identify the MPP of a limit state, (2) establish a quadratic surface around the most probable point, (3) transform the quadratic surface to a linear surface, and (4) apply FFT to provide a fast convolution solution. The method is suitable for response or performance functions that can be made approximately linear using intermediate variables.

The method for linearizing a quadratic Z-function was described in Section 3.4 for the X-based quadratic function without cross-product terms. Similarly, additional transformations such as log-transformations can be used.

Another option is to develop a full quadratic function in the u-space, then perform a coordinate transformation by a rotation to generate a new set of independent *u*-variables such that the transformed g-function becomes a quadratic function without mixed terms. Afterwards, the convolution theorem can be applied to the quadratic g(u) function. This u-based method will produce an exact SORM solution.

Assuming that the Z-function has been linearized using intermediate variables such that it can be expressed as:

$$Z(\mathbf{X}) = \sum_{i=1}^n X_i \quad (3.32)$$

The characteristic function of a random variable *X* is:

$$H_X(\omega) = \int_{-\infty}^{\infty} f_X(x) e^{j\omega x} dx \quad (3.33)$$

which is the Fourier transform of the PDF,  $f_X(x)$ . The characteristic function of a sum of independent random variables equals the product of the characteristic functions, i.e.,

$$H_Z(\omega) = \prod_{i=1}^n H_{X_i}(\omega) \quad (3.34)$$

The above equation is useful because it does not involve multiple (nested) integrations, and therefore, is particularly useful for large  $n$ . The PDF of  $Z$  is

$$f_Z(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_Z(\omega) e^{-jz\omega} d\omega \quad (3.35)$$

from which the entire CDF of  $Z$  can be computed.

Although one can use numerical integration to compute  $H_Z(\omega)$  and  $f_Z(z)$ , computational time can be substantially reduced by applying a discrete FFT scheme. The FFT requires that each random variable PDF be discretized over a sufficiently wide range. To implement the fast convolution method, the range and the number of points for FFT must be carefully considered, as discussed in Reference 3.21. Figure 3.4 illustrates the FFT procedure for  $Z = R - S$  where  $R$  and  $S$  are independent normal random variables.

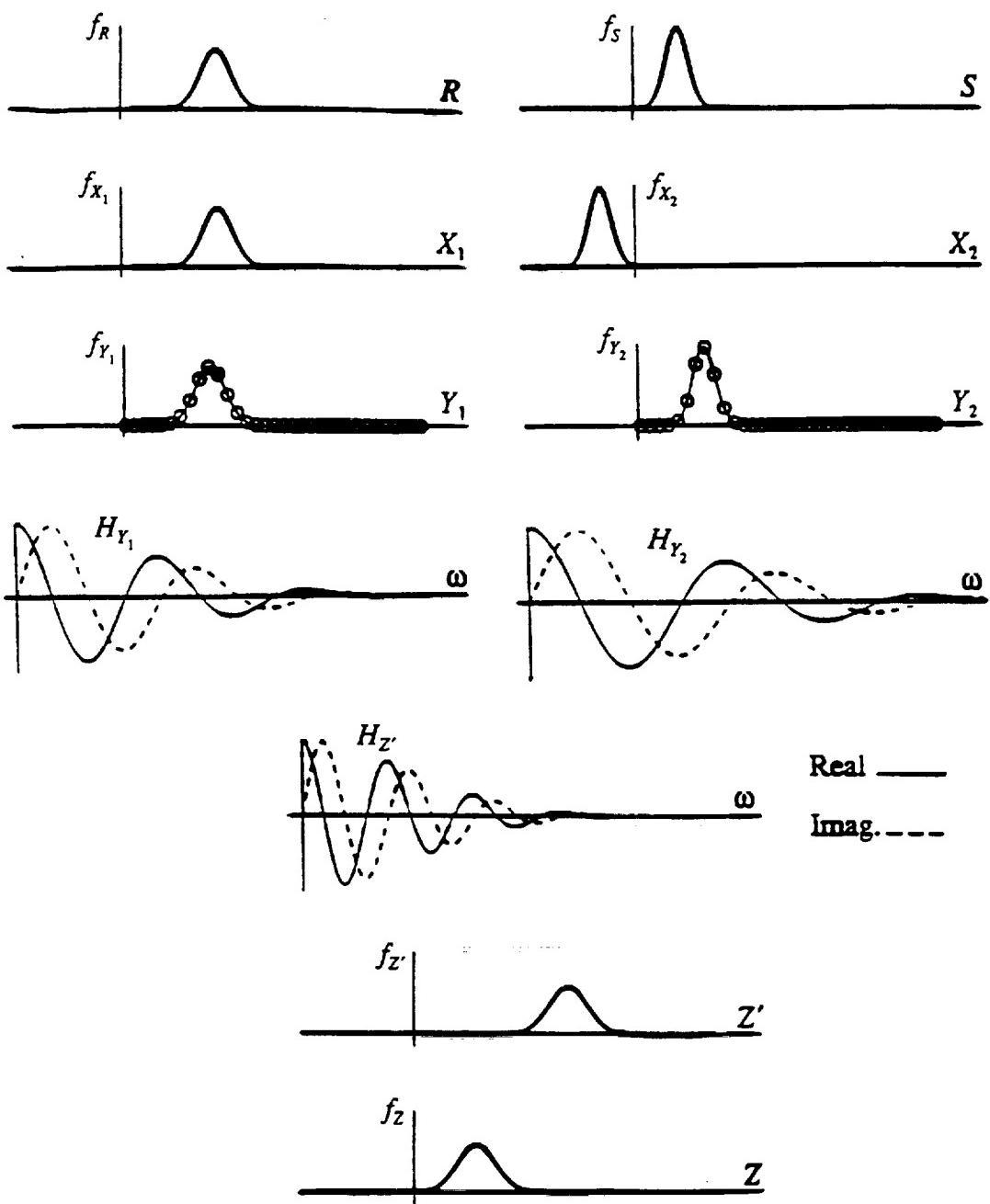


Figure 3.4 - Illustration of the Fast Convolution Procedure

### 3.6 Standard Monte Carlo Method

This method repeatedly generates a set of random values from the joint PDF,  $f_X(x)$ , and computes corresponding Z-function values. The Z-values are used to construct a CDF for Z or compute the  $p_f$  estimate for  $[Z < z_o]$ :

$$p_f = \frac{N_f}{N} \quad (3.36)$$

where  $N$  is the total number of samples and  $N_f$  is the number of samples with negative g values. Figure 3.5 illustrates a Monte Carlo procedure for a reliability analysis.

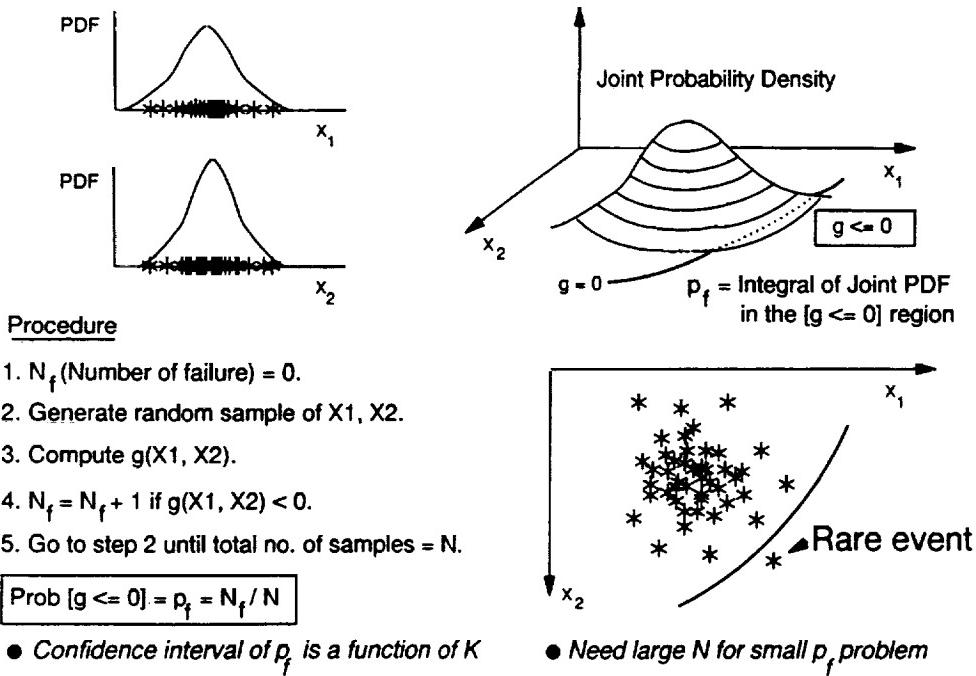


Figure 3.5 - Reliability Analysis by Monte Carlo Simulation

By specifying an error bound and a confidence interval, and assuming that the estimated probability follows a binomial distribution, the number of samples can be computed as follows:

$$\% \text{ error} = 100 \cdot \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) \sqrt{\frac{1-p}{Np}} \quad (3.37)$$

For example, for 10 % error with 95% confidence, the required number of samples is

$$N = 396 \frac{1-p}{p} \quad (3.38)$$

The probability p is unknown, but can be estimated based on a preliminary analysis with a relatively small sample size and updated with more samples.

### 3.7 Radius-based Importance Sampling

In the radius-based importance sampling methods, sampling is performed in the standardized normal ( $u$ ) space outside of a sphere (or hyper-sphere for  $n > 3$ ) as illustrated in Figure 3.6 using a sampling scheme developed by Harbitz [Reference 3.22]. For each sample generated in the  $u$ -space, the inverse transformation is used to generate a sample in the X-space for g-function evaluations. The basic assumption is that if the MPP is correct, then according to the MPP definition [ $g > 0$ ] within the sphere (even though we don't know their values). Therefore sampling is not needed within the sphere and the sampling efficiency is always better than the standard Monte Carlo method except for the special case of zero radius which is equivalent to the standard Monte Carlo method.

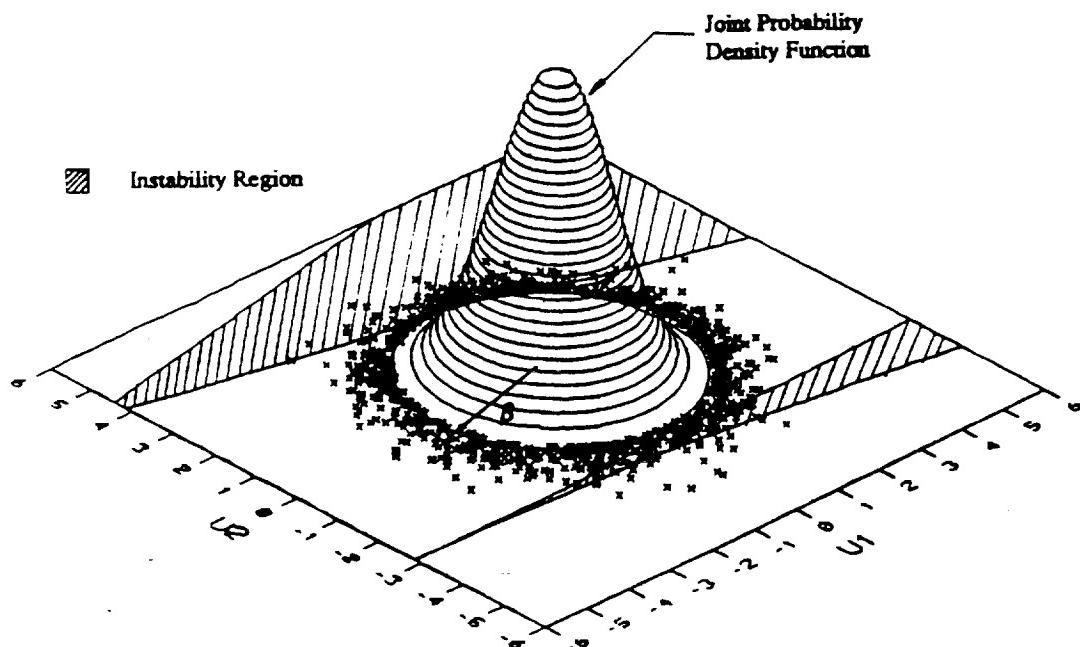


Figure 3.6 - Radius-based Importance Sampling Method

The  $p_f$  estimate is:

$$p_f = \frac{N_f}{N} \cdot p_{\text{sampling region}} \quad (3.39)$$

where  $N$  is the total number of samples (outside the sphere),  $N_f$  is the number of samples with negative g values, and  $p_{\text{sampling region}}$  is the probability outside the sphere.

NESSUS users have the options of using the radius (i.e., minimum distance) generated from the MPP or using a user-specified radius.

### 3.8 Adaptive Importance Sampling (AIS1 and AIS2)

#### Summary

The Adaptive Importance Sampling (AIS) methods minimize sampling in the safe region by adaptively and automatically adjusting the sampling space from an initial approximation of the failure region [References 3.23-3.24]. The sampling space is defined using a limit-state surface (boundary). The performance of AIS depends on the quality of the initial failure region approximation. Even though the AIS method cannot totally replace the standard Monte Carlo method, it provides an efficient accuracy improvement or checking to the MPP-based approximation methods (FORM, SORM, AMV+) for component reliability analysis. AIS also provides an efficient alternative to standard Monte Carlo for system reliability analysis.

There are two methods (AIS1, AIS2) available in NESSUS for selecting the sampling limit-state boundaries. AIS1 uses planes and AIS2 uses parabolic surfaces (see Figure 3.7). Both surfaces are constructed in the u-space and use the MPP to define the beginning sampling limit state. AIS1 changes the distance to the plane and AIS2 changes the MPP curvature. For most problems tested to date, both methods performed equally well with AIS2 having a slight edge. AIS1 is less sensitive to the accuracy of the initial MPP but may require more samples. AIS2 is more sensitive to the accuracy of the initial MPP but is generally more efficient.

For user-defined g-functions, AIS is highly recommended because of its overall effectiveness in accuracy and efficiency. It should be cautioned, however, that the performance of AIS depends on the accuracy of the MPP. If the starting MPP is far away from the true MPP, AIS may either require a large number of samples than usually required or may miss a failure region. Therefore, if NESSUS fails to find the MPP, AIS may not be efficient or may converge incorrectly.

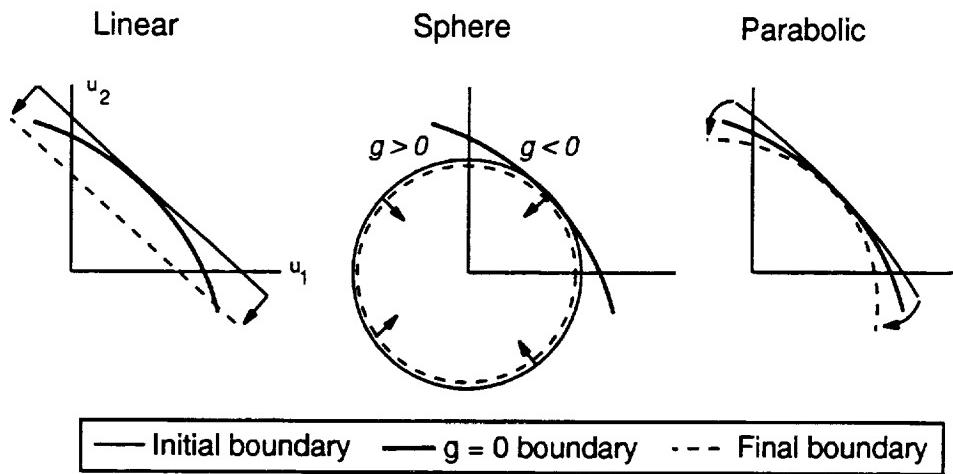


Figure 3.7 - Illustration of the AIS Concept

### AIS Methodology

Consider first a single limit state. The AIS approach has the following features: (1) the sampling density is proportional to the joint PDF of the random variables, (2) the initial sampling domain is the failure side of the limit state surface constructed from an approximate (e.g., parabolic) surface, (3) additional samples are added after incrementally increasing the sampling region (e.g., by changing the curvatures of the parabolic surface at the most probable point), and (4) the final sampling domain contains, but is only slightly greater than, the failure domain. The objective is to minimize over-sampling in the safe region. Sampling in the safe region is needed only to identify the limit state surface.

To minimize sampling, a good adaptive surface should be able to represent the exact limit-state surface closely. Compatible with the MPP-based methods, there are several possibilities in choosing and varying the sampling limit state surface as illustrated in Figure 3.7, where three kinds of sampling boundary surfaces are shown.

The radius or sphere-based method is the simplest to implement and is the most robust because the initial MPP is only used to define an initial sampling radius. The disadvantage is that the sampling efficiency (related to the probability inside the sphere) decreases as the number of random variables,  $n$ , increases.

The plane-based method is also easy to implement and the sampling efficiency depends on the minimum distance and is not affected by  $n$ . However, the initial MPP is important in this method and the method is therefore not as robust as the sphere-based method and not as efficient as the curvature-based method.

The curvature-based method adaptively changes the curvature of the parabolic surface at the MPP. The method's efficiency and robustness relies on the accuracy of the initial MPP but is generally the most efficient among the three methods because of it's ability to closely envelop the failure region. The curvature-based AIS concept is further illustrated in Figure 3.8, in which the parabolic surface is rotationally symmetric about the vector OP that passes through the origin and the MPP. The sampling domain can be adjusted by increasing or decreasing the curvatures of the parabolic surface. By decreasing the curvatures to the extreme ( $-\infty$ ), the sampling region will cover the entire space. However, if  $\beta$  is correctly calculated, sampling inside the sphere is not needed. Therefore, the AIS2 bound is a sphere, as illustrated in Figure 3.8. For additional checking, the distance to the MPP can also be reduced in case the calculated  $\beta$  is in error.

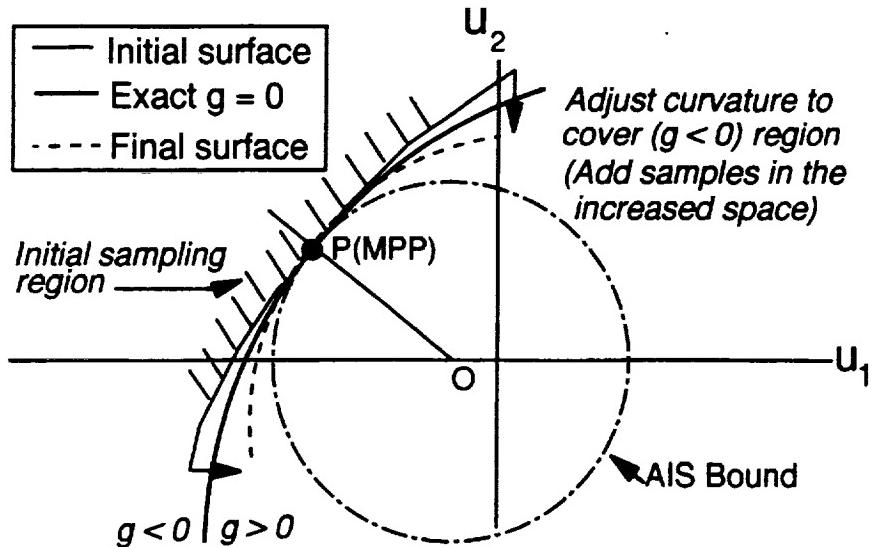


Figure 3.8 - Curvature-based AIS Mehtod

To describe how the curvature-based AIS method is related to other NESSUS methods (AMV+, FORM, and SORM), assume that an initial second-order limit state is available in the form

$$g(\mathbf{u}) = \nabla g(\mathbf{u}^*)^T (\mathbf{u} - \mathbf{u}^*) + \frac{1}{2} (\mathbf{u} - \mathbf{u}^*)^T \mathbf{H}(\mathbf{u}^*) (\mathbf{u} - \mathbf{u}^*) \quad (3.40)$$

where  $\nabla g(\mathbf{u}^*)$  is the gradient at the MPP, and  $\mathbf{H}$  is the Hessian matrix containing second-order derivatives. Because Equation 3.40 will be used to define an initial sampling domain for the AIS, only a rough estimate is needed. A linear  $g(\mathbf{X})$ -function generated by the AMV+ method (See Section 3.9) can be transformed to the  $\mathbf{u}$ -space and used to generate a second-order

$g(\mathbf{u})$ -function. The transformation procedure to develop a parabolic surface is well documented in the literature [Reference 3.16] and is given by the equation

$$g = \beta - v_n + \sum_{i=1}^{n-1} \lambda_i v_i^2 \quad (3.41)$$

in which  $v_i$  are independent standardized normal variables, and  $\lambda_i$  are related to the main curvatures by the relation  $\kappa_i = 2\lambda_i$ . The curvature-based AIS procedure uses Equation 3.41 to develop the initial adaptive sampling boundary and generate the samples in the  $[g < 0]$  domain. The corresponding probability in the sampling region,  $p_s$ , can be numerically computed [Reference 3.25].

To change the sampling space, the  $\lambda_i$  are changed to  $\lambda'_i$ . The changes in  $\lambda_i$  can be made individually or simultaneously based on a selected probability increment  $\Delta p_s$ . Define the changed or perturbed limit state surface as

$$g' = \beta + \sum_{i=1}^{n-1} \lambda'_i v_i^2 \quad (3.42)$$

where the superscript (prime) indicates a perturbed condition.

Let  $N_1$  be the initial number of samples determined based on a confidence interval and an error bound. During the sampling process, this number will be adjusted based on the updated probability of failure in the sampling region. The number of failure points in  $p_s$  is denoted as  $N_1^+$ . Given a set of  $\lambda'_i$ , the sampling is applied to an increased sampling  $\Delta S$  region. A predetermined number of samples,  $\Delta N$ , in the  $\Delta S$  region, is calculated using

$$\Delta N = (p_s' - p_s)N_1 = \Delta p_s N_1 \quad (3.43)$$

By computing the original  $g$ -function, the number of failure points,  $\Delta N^+$ , within  $\Delta S$ , can be determined and the updated probability estimate becomes

$$\hat{p} = p_s' \cdot \frac{N_1^+ + \Delta N^+}{N_1 + \Delta N} \quad (3.44)$$

The perturbation procedure can be repeated until no more failure points are observed after at least one perturbation. A more detailed computational procedure is given in Reference 3.26.

Assuming that the sampled  $p$  estimate has a normal distribution, for a  $(1 - \alpha)$  confidence interval, the error bound,  $\gamma$  (in percentage), is given by

$$\gamma = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) \sqrt{\frac{1 - \hat{p}}{\hat{p}N}} \quad (3.45)$$

where  $\Phi(\cdot)$  is the standard normal distribution function and  $\hat{p}$  is the probability of failure estimate in the importance sampling region. Equation 8.6 can be used to calculate the required samples,  $N$ , given an estimated  $\hat{p}$ . Ideally, we wish to apply AIS to a region very close to the exact failure region resulting in  $\hat{p} = 1$ , and  $N = 0$ . This, however, can never be achieved because the sampling region must be greater than the failure region to indicate that the failure region has been covered sufficiently. In addition, the parabolic surface may not be a sufficiently flexible surface to approximate the true surface.

The convergence procedure implemented in the NESSUS code is as follows:

- (1) Generate samples in the initial sampling domain  $S$ . Begin with a small sample (10 points) to estimate  $\hat{p}$ . Then gradually increase the samples until the error bound  $\gamma$  is acceptable for a selected confidence level.
- (2) Reduce the curvatures (average curvatures are used in NESSUS) such that the increased probability  $\Delta p$  is a fraction (10%) of the initial probability  $p$ . Generate samples only in the increased domain  $S$ .
- (3) Update the probability estimate. Stop the analysis if the probability estimate converges. Otherwise, decrease the curvature again.
- (4) In case  $\beta$  is in error, slightly reduce the minimum distance (i.e., shift the parabolic surface towards the origin) and generate additional samples. Stop the analysis if no more failure points are observed.

### AIS for System Reliability

In NESSUS a system failure is defined using a fault tree, which provides a way to manage multiple failure modes. The limit state functions are defined in the bottom events. Sequential failures can be modeled using the PRIORITY AND gate. A sequence of g-functions, corresponding to a sequence of updated structural configurations with load redistribution can be explicitly or implicitly defined in the bottom events.

Through a fault tree, all the failure modes can be defined. A failure mode can involve one or more limit states. By adding all the failure modes, and therefore all the limit states, the system limit state surface can be constructed piece by piece. The AIS procedure for system reliability analysis requires the construction of multiple parabolic surfaces. In principle, it is a

straightforward extension of the concept for one limit state. The difficult part is to develop a procedure to add failure regions.

The approach implemented in NESSUS adds samples progressively starting from the most important limit state, based on the approximate functions. This approach is ineffective for cases where system failure is governed by the joint effects from several limit states. In such cases, no limit state can be considered as dominate because the MPP of the individual limit state is not a likely event for a system failure. A more effective computational procedure adds samples progressively based on failure modes. The initial sampling regions are based on the approximate limit states generated by the AMV+ method (Section 3.9) and the increased sampling regions are based on the perturbed limit states.

Figure 3.9 illustrates the above curvature-based AIS procedure for a system with two failure modes that involve three limit states. The probability of failure statement is

$$p_f = P\{(g_1 < 0) \cup [(g_2 < 0) \cap (g_3 < 0)]\} . \quad (3.46)$$

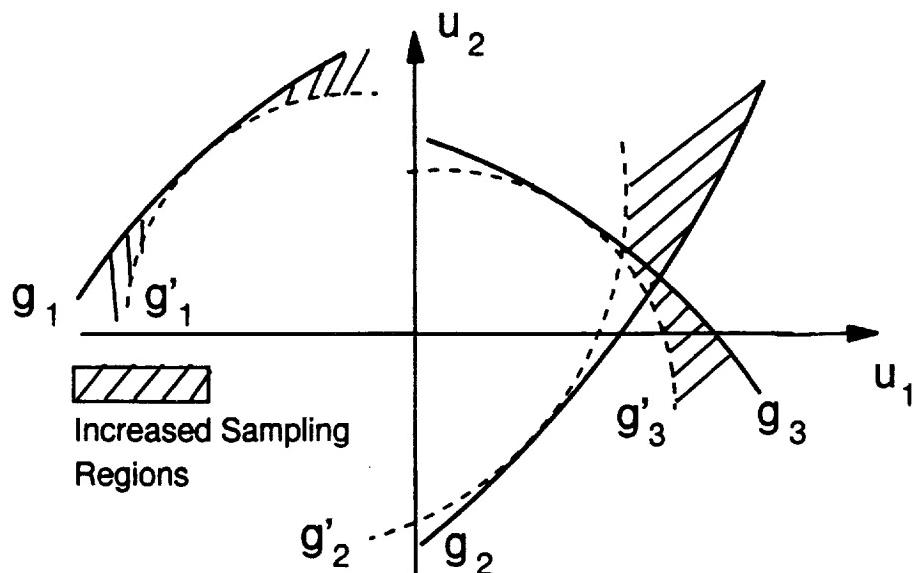


Figure 3.9 - AIS for System Reliability

The above adaptive approach for system reliability can be easily modified for use with the radius-based and plane-based methods. However, the curvature-based AIS is recommended for its overall performance in efficiency and robustness.

### Reliability Sensitivity with Respect to a Distribution Parameter

The AIS-based sensitivity analysis, not automated in the current version of NESSUS, can be performed using the AIS-generated samples.

When a distribution parameter is changed, the sensitivity of the probability of failure (denoted as  $p$  for simplicity) with respect to a distribution parameter,  $\theta$ , can be evaluated using

$$\frac{dp}{d\theta} = \int_{\Omega} \dots \int \frac{df_x}{d\theta} dx \quad (3.47)$$

Therefore,

$$\frac{dp/p}{d\theta/\theta} = \int_{\Omega} \dots \int \frac{\theta}{p f_x d\theta} \frac{df_x}{dx} f_x dx = \int_{\Omega} \dots \int \frac{\theta df_x f_x}{f_x d\theta p} dx \quad (3.48)$$

or

$$\frac{d \ln p}{d \ln \theta} = E \left[ \frac{d \ln f_x}{d \ln \theta} \right]_{\Omega} \quad (3.49)$$

where the expected value is evaluated using the PDF in the failure region. Because the AIS approach uses the same PDF, the probability sensitivity can be computed using those AIS points in the failure region. No additional g-function calculations are required.

Two useful dimensionless probability sensitivity coefficients,  $S$ , are

$$S_{\sigma} = \frac{d \ln p}{d \ln \sigma} \quad (3.50)$$

$$S_{\mu} = \frac{d \ln p}{d \mu / \sigma} \quad (3.51)$$

For some random variables, (e.g., independent normal and lognormal variables), these coefficients can be analytically derived. For example, let  $\mu_i$  and  $\sigma_i$  be the mean and the standard deviation, respectively, of the normal random variable  $i$ . It can be shown that

$$\frac{d \ln p}{d \ln \sigma_i} = E[u_i^2 - 1]_{\Omega} \quad (3.52)$$

and

$$\frac{d \ln p}{d \mu_i / \sigma_i} = \frac{1}{\sigma_i} E[u_i]_{\Omega} \quad (3.53)$$

Assuming that a small change in  $\theta$ , denoted as  $\Delta\theta$ , results in a small change in  $p$ ,  $\Delta p$ , the changes in  $p$  can be approximated by

$$\Delta p = p \cdot S \cdot \frac{\Delta\theta}{\sigma} \quad (3.54)$$

A random variable approaches deterministic if  $\sigma$  approaches 0, i.e.,  $\Delta\sigma = -\sigma$ . In this case,

$$\Delta p = -p S_{\sigma} \quad (3.55)$$

which suggests that  $S_{\sigma}$  can be used to rank the importance of the random variable uncertainty ( $\sigma_i$ ). Similarly, if we reduce  $\mu$  by one  $\sigma$  for a random variable,

$$\Delta p = -p S_{\mu} \quad (3.56)$$

Therefore,  $S_{\mu}$  can be used to rank the importance of the uncertainty in the mean values.

The above AIS-based sensitivity analysis approach is particularly useful for system reliability problems in which there are multiple MPP's. With the AIS method, the sensitivity calculations are straightforward.

### 3.9 Advanced Mean Value Methods (MV, AMV, AMV+)

The methods described here have been developed primarily for complicated g-functions that require time-consuming calculations. For simple g-functions, these methods may not offer time savings.

#### Mean Value Method

Assume that the Z-function is smooth and Taylor's series expansion of Z exists at the mean values. The Z-function can be expressed as:

$$\begin{aligned} Z(\mathbf{X}) &= Z(\boldsymbol{\mu}) + \sum_{i=1}^n \left( \frac{\partial Z}{\partial X_i} \right) \cdot (X_i - \mu_i) + H(\mathbf{X}) \\ &= a_o + \sum_{i=1}^n a_i X_i + H(\mathbf{X}) \\ &= Z_{MV}(\mathbf{X}) + H(\mathbf{X}) \end{aligned} \quad (3.57)$$

where the derivatives are evaluated at the mean values;  $Z_{MV}$  is a random variable representing the sum of the first-order terms and  $H(\mathbf{X})$  represents the higher-order terms.

There are several ways of obtaining  $a_i$ . In NESSUS, the coefficients  $a_i$  are computed by numerical differentiation method (for user-defined g-functions) or by the least-squares method (for user-defined data sets). The minimum required number of Z-function evaluations is  $(n+1)$  using a numerical differentiation method.

By retaining only the first-order terms, the mean and the standard deviation of  $Z$ , assuming independent random variables, are approximately:

$$\mu_Z \approx a_o + \sum_{i=1}^n a_i \mu_{X_i} \quad (3.58)$$

$$\sigma_Z^2 \approx \sum_{i=1}^n a_i^2 \sigma_{X_i}^2 \quad (3.59)$$

These two statistical moments can be computed easily after the coefficients  $a_i$  are computed. Equations 3.58 and 3.59 constitute the mean-based first-order, second-moment method. However, when the probability distributions, not just the first two moments, of  $X_i$  are fully defined, the CDF of the first-order terms,  $Z_{MV}$ , is also fully defined. Since the  $Z_{MV}$ -function is linear and explicit, its CDF can be computed effectively using many methods. Thus, the mean-value first-order (MVFO, abbreviated as MV) solution defines the CDF of  $Z_{MV}$ , not just the two moments.

For nonlinear Z-functions, the MV solution is, in general, not sufficiently accurate. For simple problems, it is possible to use higher-order expansions to improve the accuracy. For example, a mean-value second-order solution can be obtained by retaining second-order terms in the series expansion. However, for problems involving implicit Z-functions and large  $n$ , the higher-order approach becomes difficult and inefficient. The AMV method described below provides an alternative to improve the MV solution with minimum additional Z-function evaluations.

#### Advanced Mean Value Method

The advanced mean-value first-order (AMVFO, abbreviated as AMV) method blends the conventional mean value method with the NESSUS concept to compute the CDF. The AMV method improves the mean value method by using a simple correction procedure to compensate for the errors introduced from the truncation of a Taylor's series. In addition, the AMV-based CDF provides information on the nonlinearity of the g-function to detect potential numerical problems [Reference 3.28].

The AMV model is defined as:

$$Z_{AMV} = Z_{MV} + H(Z_{MV}) \quad (3.60)$$

where  $H(Z_{MV})$  is defined as the difference between the values of  $Z_{MV}$  and  $Z$  calculated at the Most Probable Point Locus (MPPL) of  $Z_{MV}$ . The MPPL is defined by connecting all the MPP's for different  $z$  values [Reference 3.29]. The MPPL of  $Z_{MV}$  is generally a nonlinear curve in the  $u$ -space. For a complicated structural analysis, the construction of  $Z_{MV}$  may be time-consuming, but its MPPL can be computed easily.

The key to the AMV method is the reduction of the truncation error by replacing the higher-order terms  $H(\mathbf{X})$  by a simplified function  $H(Z_{MV})$  dependent on  $Z_{MV}$ . Ideally, the  $H(Z_{MV})$  function should be based on the exact most-probable-point locus (MPPL) of the  $Z$ -function to optimize the truncation error [References 3.28-3.29]. The AMV procedure simplifies this procedure by using the MPPL of  $Z_{MV}$ . As a result of this approximation, the truncation error is not optimum; however, because the  $Z$ -function correction points are generally "close" to the exact most probable points, the AMV solution provides a reasonably good CDF estimation for many engineering application problems.

The computational steps for a point CDF analysis are: (1) Based on  $Z_{MV}$ , compute the MPP,  $\mathbf{x}^*$ , for a selected CDF value. (2) Compute  $Z(\mathbf{x}^*)$  to update  $z$  for the selected CDF value. Given the MV model, the required number of  $Z$ -function calculations equals the number of selected CDF values. The above steps require the construction of the  $Z_{MV}$ -function only once for all the CDF levels. Assuming that a numerical differentiation scheme is used to define the  $Z_{MV}$ -function, the required number of the  $Z$ -function evaluations is  $(n + 1 + m)$ , where  $n$  is the number of random variables and  $m$  is the number of CDF levels. Figure 3.10 illustrates the above AMV procedure for a selected CDF. The accuracy of the CDF solution depends on the quality of the MPPL from  $Z_{MV}$ , i.e., the solution is good if the approximate locus is close to the exact one.

#### Advanced Mean Value Plus (AMV+) Method

In general, the AMV solution can be improved by using an improved expansion point, which can be done typically by an optimization procedure or an iteration procedure. Based initially on  $Z_{MV}$  and by keeping track of the MPPL [Reference 3.14], the exact MPP for a limit state  $Z = z$  can be computed to establish the AMV+ model defined as:

$$Z_{AMV+} = Z(\mathbf{x}^*) + \sum_{i=1}^n \left( \frac{\partial Z}{\partial X_i} \right) \Big|_{\mathbf{x}^*} (X_i - x_i^*) = b_o + \sum_{i=1}^n b_i X_i \quad (3.61)$$

where  $\mathbf{x}^*$  is the converged MPP for  $Z = z$ .

$$Z_{AMV} = Z_{MV} + H(Z_{MV})$$

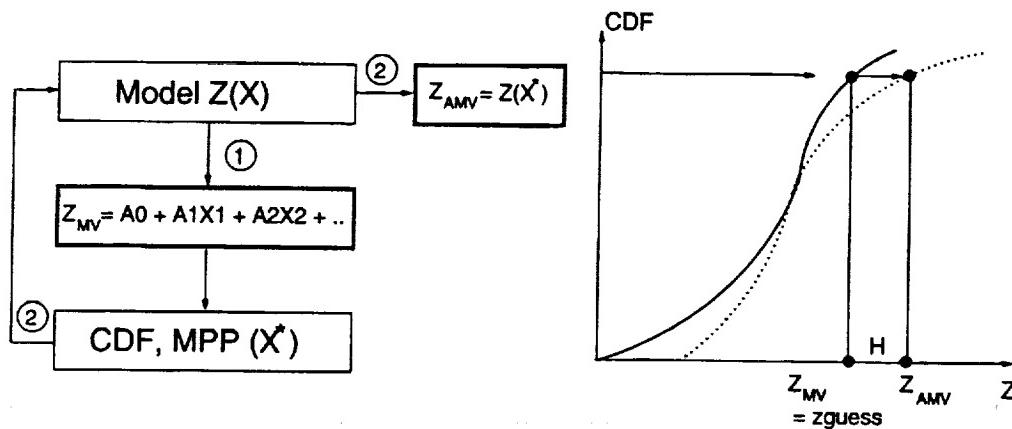


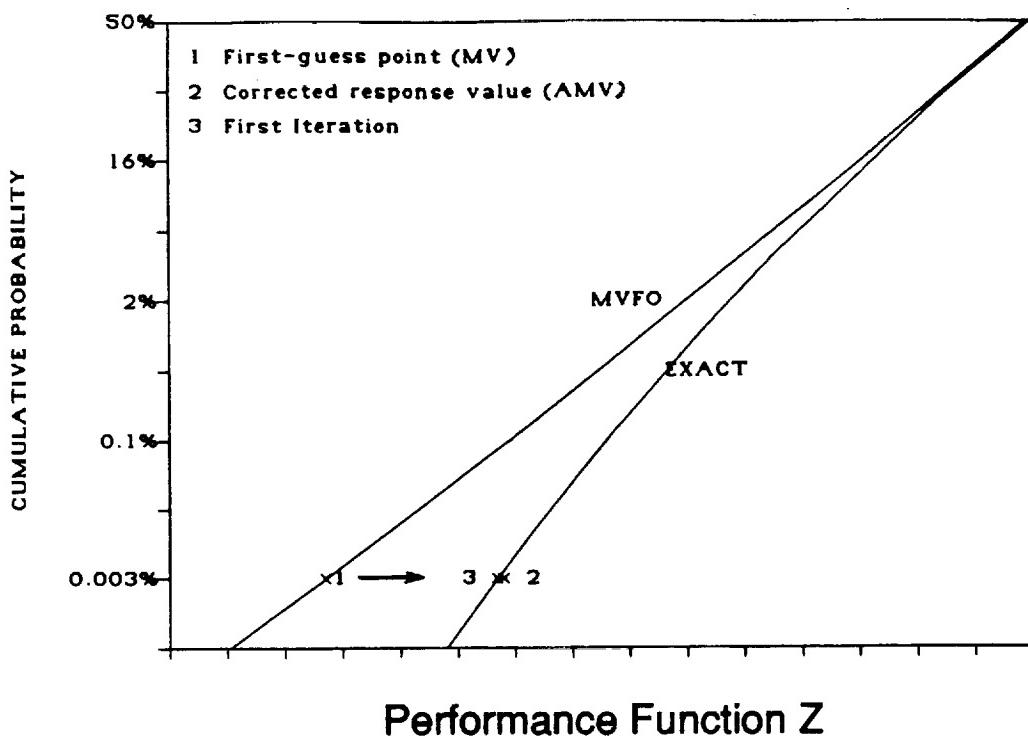
Figure 3.10 - Illustration of the AMV Method

It is important to note that the variables  $X_i$  are generally non-normal and dependent, therefore the above AMV+ model, which is linear in the  $X$ -space, is different from the first-order reliability method (FORM) model, which is linear in the  $u$ -space. An advantage of Equation 3.61 is that it provides an initial estimate of the curvature (about the MPP) in the  $u$ -space that can be used both for approximate probability of failure analysis and for starting the AIS2 analysis described in Section 3.8. It is particularly useful when the curvature is primarily caused by the non-normal to normal transformation.

Based on the AMV results, two iteration algorithms, one for specified probability level and the other for specified  $Z$  level have been proposed [References 3.9, 3.28] to improve the CDF estimates. The two algorithms are illustrated in Figures 3.11-3.12. The algorithm for specified probability level is summarized in the following steps:

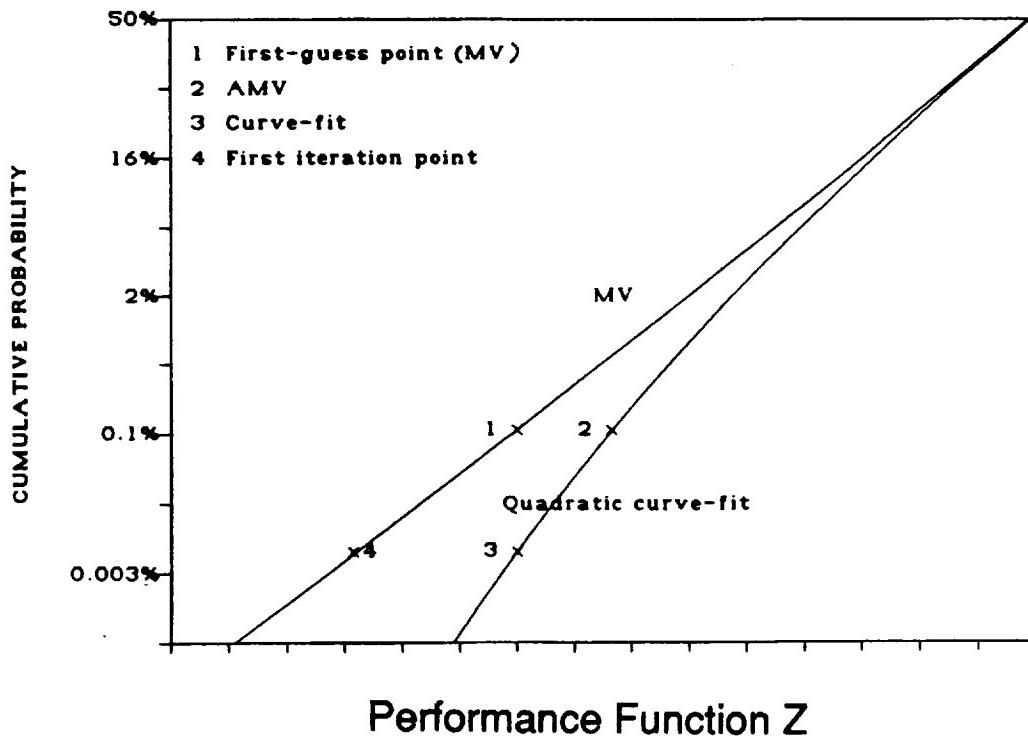
- Construct a linear  $Z$ -function,  $Z_1$ , initially mean-based, and search for  $z_o$  such that  $P[Z_1 < z_o] = \text{probability goal}$ .
- Use most probable point of  $Z_1 = z_o$  and re-compute  $Z$ .
- Obtain the new  $Z_1$ -function around the most probable point of  $Z_1 = z_o$ .
- Repeat the above steps until  $z_o$  converges.

In general, the above steps require the construction of the  $Z_1$ -function several times. Therefore, for complicated  $Z$ -functions which require extensive computations, efficient sensitivity computation schemes are preferred in updating the  $Z_1$ -function.



**Performance Function Z**

Figure 3.11 - AMV+ Iteration Algorithm for a Specified P-level



**Performance Function Z**

Figure 3.12 - AMV+ Iteration Algorithm for a Specified Z-level

### Example

To illustrate the AMV and AMV+ method, consider an example where the Z-function is the first bending natural frequency of a cantilever beam. This frequency can be approximated as

$$f_n = 0.5602 \sqrt{\frac{Et^2}{12\rho L^4}} \quad (3.62)$$

where E is the modulus of elasticity, t is the thickness,  $\rho$  is the material density, and L is the length. The random variables are defined in Figure 3.13.

By numerical differentiation at the mean values of the random variables, the following first-order approximation can be obtained using the results of five Z-function (i.e., Equation 3.62) evaluations.

$$f_{MV} = a_0 + a_1 E + a_2 t + a_3 \rho + a_4 L \quad (3.63)$$

Based on Equation 3.63 an MV CDF solution can obtained as shown in Figure 3.62. Note that the CDF solution is plotted on a normal probability paper.

The above MV solution is exact if  $f_n$  is a linear function of the four random variables. However, since Equation 3.62 is a non-linear function, Equation 3.63 is subjected to error in the regions away from the mean values. For each selected probability, the AMV solution is obtained by calculating  $f_n$  (Equation 3.62) at the most probable point ( $E^*$ ,  $t^*$ ,  $\rho^*$ ,  $L^*$ ) obtained using Equation 3.63. This requires one function evaluation of  $f_n$  for each selected probability.

In Figure 3.13, three points are shown at a selected probability level (CDF = 0.003%). Point 1 is the MV solution, point 2 is the AMV solution, and point 3 is the AMV+ solution.

### AMV-based Methods (AMV and AMV+) for Non-monotonic g-functions

The previous AMV-based methods have been applied to numerous problems to validate the procedure and code [References 3.28, 3.30, 3.31]. In those tested examples, the AMV-based method provided satisfactory solutions. When the Z-function is a strongly non-monotonic function of  $X$ , the AMV-based CDF solution may suggest that the Z-function is a strongly non-monotonic function of  $Z_{MV}$ . Based on this information, modified CDF solutions has been developed [Reference 3.28].

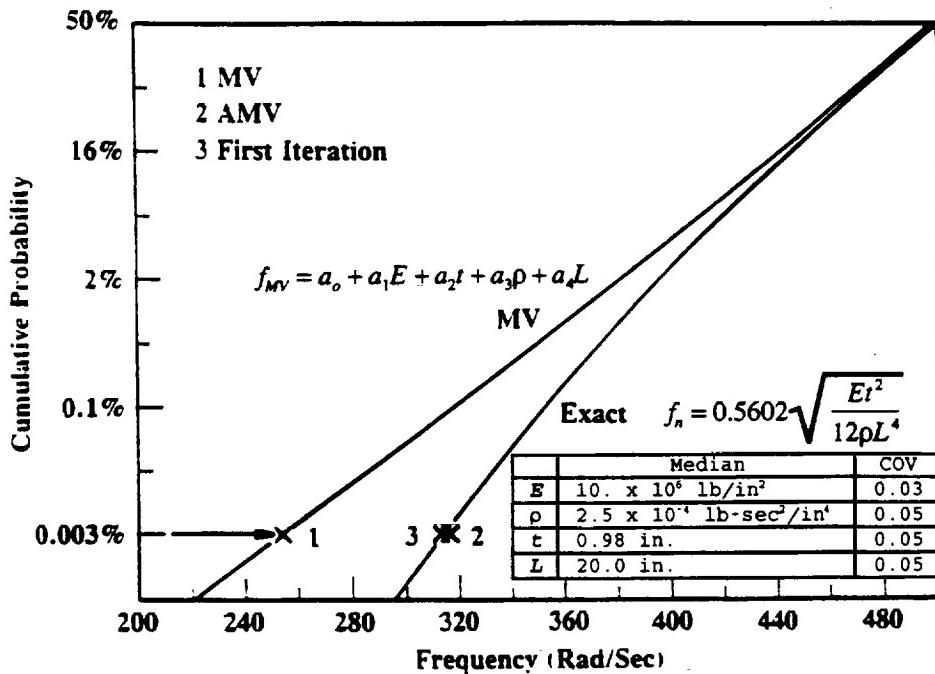


Figure 3.13 - The AMV+ Analysis for the Cantilever Beam Example [Reference 3.28]

### 3.10 System Reliability Analysis (Fault Tree Method)

A system reliability problem is one that involves multiple limit states due to multiple components and/or multiple failure modes. In NESSUS, a system failure is defined using a fault tree, which provides a systematic way to manage multiple failure modes. A fault tree has three major characteristics; bottom events, combination gates, and the connectivity between the bottom events and gates. NESSUS is presently limited to AND and OR gates. The limit state functions are defined in the bottom events. Sequential failures can be modeled using the PRIORITY AND gate. A sequence of g-functions, corresponding to a sequence of updated structural configurations with load redistribution can be explicitly or implicitly defined in the bottom events as illustrated in Figure 3.14.

Through a fault tree, all the failure modes can be defined. A failure mode can involve one or more limit states. By adding all the failure modes, and therefore all the limit states, the system limit state surface can be constructed piece by piece.

There are several methods for system reliability calculations. A common approach is based on the bounding theory [References 3.3, 3.4, 3.32], which provides reliability bounds for serial, parallel, and combined systems. These methods provide approximate solutions.

In NESSUS, system reliability is computed using the adaptive importance sampling method described in Section 3.8.

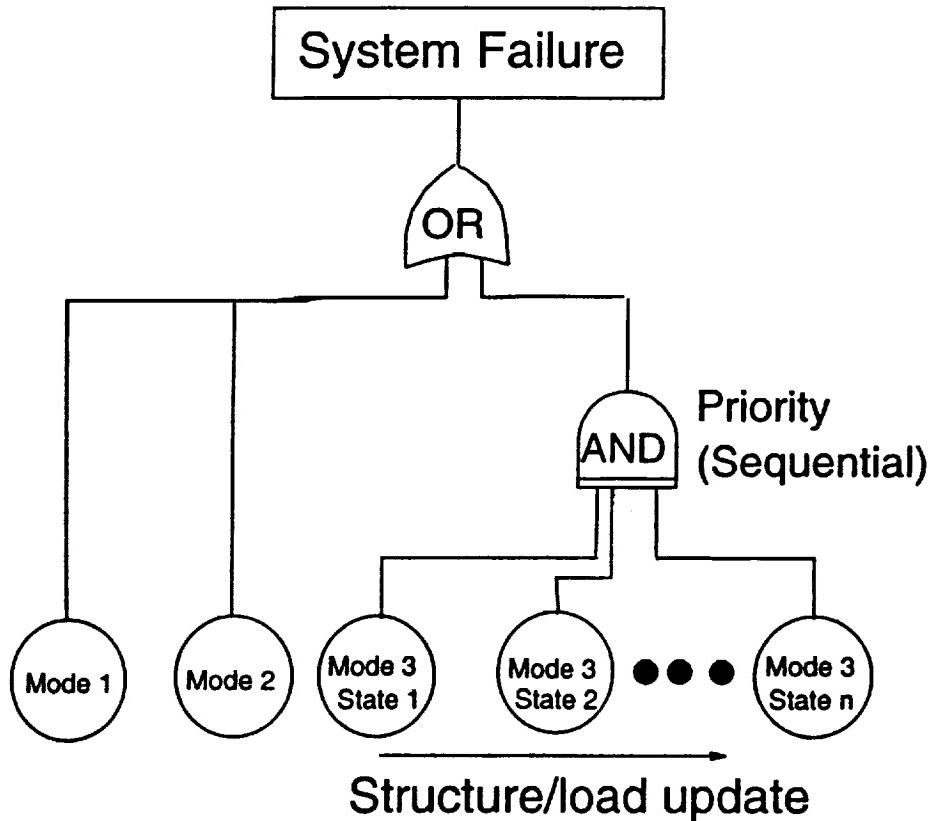


Figure 3.14 - Probabilistic Fault Tree Approach for System Reliability Analysis

## References

- 3.1 CIRIA, *Rationalization of Safety and Serviceability Factors in Structural Codes*, Construction Industry Research and Information Association, Report 63, 6 Storey's Gate, London SWIP 3AU, July, 1977.
- 3.2 Leporati, E., *The Assessment of Structural Safety*, Research Studies Press, 1979.
- 3.3 Ang, A. H.-S., and Tang, W. H., *Probability Concepts in Engineering Planning and Design, Volume II: Decision, Risk, and Reliability*, John Wiley & Sons, Inc., New York, 1984.
- 3.4 Madsen, H. O., Krenk, S., and Lind, N. C., *Methods of Structural Safety*, Prentice Hall Inc., Englewood Cliffs, New Jersey, 1986.
- 3.5 Rosenblatt, M., "Remarks on a Multivariate Transformation," *The Annals of Mathematical Statistics*, Vol. 23, No. 3, pp. 470-472, 1952.

- 3.6 Hohenbichler, M., and Rackwitz, R., "Non-normal Dependent Vectors in Structural Safety," *Journal of Engineering Mechanics Division*, ASCE, Vol. 100, No. EM6, pp. 1227-1238, Dec. 1981.
- 3.7 Grigoriu, M., "Approximate analysis of complex reliability problems," *Structural Safety*, Vol. 1, pp. 277-288, 1983.
- 3.8 Der Kiureghian, A., and Liu, P.-L., "Structural reliability under incomplete probability information," *Journal of Engineering Mechanics*, ASCE, Vol. 112, No. 1, pp. 85-104, 1986.
- 3.9 Wu, Y.-T., Burnside, O. H., and Cruse, T. A., "Probabilistic Methods for Structural Response Analysis," Computational Mechanics of Probabilistic and Reliability Analysis, edited by W.K. Liu and T. Belytschko, Elmepress International, Chap. 7, 1989. (Presented at the 1988 ASME Summer Meeting and published in *Computational Probabilistic Methods*, W.K. Liu et al., eds., AMD-Vol. 93, ASME, pp. 1-14, 1988.)
- 3.10 Rackwitz, R., and Fiessler, B., "Structural Reliability Under Combined Load Sequences," *Journal of Computers and Structures*, Vol. 9, pp. 489-494, 1978.
- 3.11 Ditlevsen, O., "Principle of Normal Tail Approximation," *Journal of Engineering Mechanics Division*, ASCE, Vol. 107, pp. 1191-1208, Dec. 1981.
- 3.12 Liu, P.-L., and Der Kiureghian, A., "Optimization Algorithms for Structural Reliability Analysis," Report No. UCB/SESM-86/09, Dept. of Civil Engineering, Univ. of California, Berkeley, July 1986.
- 3.13 Wu, Y.-T., Torng, T. Y., and Khalessi, M. R., "A New Iteration Procedure for Efficient Structural Reliability Analysis," *Proceedings of ISUMA '90, the First International Symposium on Uncertainty Modeling and Analysis*, IEEE Computer Society Press, pp. 72-77, Dec. 1990.
- 3.14 Wu, Y.-T., Gureghian, B., Sagar, B., and Codell, R. B., "Sensitivity and Uncertainty Analysis Applied to One-Dimensional Transport in a Layered Fractured Rock. Part II: Probabilistic Methods Based on the Limit-State Approach", *Nuclear Technology Journal*, Vol. 104, No. 2, pp. 297-308, Nov. 1993.
- 3.15 Fiessler, B., Neumann, H. J., and Rackwitz, R., "Quadratic Limit States in Structural Reliability," *Journal of Engineering Mechanics*, ASCE, Vol. 105, Aug. pp. 661-676, 1979.
- 3.16 Tvedt, L., "Distribution of Quadratic Forms in Normal Space — Application to Structural Reliability," *Journal of Engineering Mechanics*, Vol. 116, No. 6, pp. 1183-1197, 1990.

- 3.17 Breitung, K., "Asymptotic Approximation for Multi-normal Domain and Surface Integrals," Fourth International Conference on Application of Statistic and Probability in Soil and Structural Engineering, University di Firenze, Pitagora Editrice, 1983.
- 3.18 Chen, X., and Lind, N. C., "Fast Probability Integration by Three-Parameter Normal Tail Approximation," *Structural Safety*, Vol. 1, pp. 269-276, 1983.
- 3.19 Wu, Y.-T., "Demonstration of a New Fast Probability Integration Method for Reliability Analysis," *Journal of Industry*, ASME, pp. 24-28, Feb. 1987.
- 3.20 Wu, Y.-T., and Wirsching, P. H., "New Algorithm for Structural Reliability Estimation," *Journal of Engineering Mechanics*, ASCE, Vol. 113, No. 9, pp. 1319-1336, Sept. 1987.
- 3.21 Wu, Y.-T., and Tornig, T. Y., "A Fast Convolution Procedure for Probabilistic Engineering Analysis," Proceedings of *ISUMA '90, the First International Symposium on Uncertainty Modeling and Analysis*, IEEE Computer Society Press, pp. 670-675, Dec. 1990.
- 3.22 Harbitz, A., "An Efficient Sampling Method for Probability of Failure Calculation," *Structural Safety*, Vol. 3, pp. 109-115, 1986.
- 3.23 Wu, Y.-T., and Burnside, O., H., "Computation of Probability of Instability using the Fast Probability Integration Method," Presented at the ASCE Specialty Conference on Probabilistic Methods, May 1987.
- 3.24 Wu, Y.-T., Tornig, T. Y., Burnside, O. H., and Rheinfurth, M. H., "Probabilistic Rotordynamics Analysis Using an Adaptive Importance Sampling Method," *Proceedings of the ASCE Specialty Conference on Probabilistic Mechanics, and Structural and Geotechnical Reliability*, 1992.
- 3.25 Bjerager, P., "Probability Integration by Directional Simulation," *Journal of Engineering Mechanics*, Vol. 114, No. 8, pp. 1285-1302, 1987.
- 3.26 Wu, Y.-T., "Computational Methods for Efficient Structural Reliability and Reliability Sensitivity Analysis" *AIAA Journal*, Vol. 32, No. 8, Aug. 1994, pp. 1717-1723. Presented at the 34th SDM Conference, 1993.
- 3.27 Wu, Y.-T., "Adaptive Importance Sampling (AIS)-based System Reliability Sensitivity Analysis," *Proceedings of the IUTAM Symposium on Probabilistic Structural Mechanics: Advances in Structural Reliability Methods*, Eds. P.D. Spanos and Y.-T. Wu; Springer-Verlag, pp. 550-564, Oct. 1994.
- 3.28 Wu, Y.-T., Millwater, H. R., and Cruse, T. A., "An Advanced Probabilistic Structural Analysis Method for Implicit Performance Functions," *AIAA Journal*, Vol. 28, No. 9, pp. 1663-1669, 1990.

- 3.29 Wu, Y.-T., Burnside, O. H., and Dominguez, J., "Efficient Probabilistic Fracture Mechanics Analysis," *Proceedings of the 4th International Conference on Numerical Methods in Fracture Mechanics*, Pineridge Press, Swansea, U. K., pp. 85-100, 1987.
- 3.30 Wu, Y.-T., and Burnside, O. H., "Validation of the NESSUS Probabilistic Analysis Computer Program," *Proceedings of the 29th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference*, Part 3, pp. 1267-1274, 1988.
- 3.31 Cruse, T. A., Wu, Y.-T., Dias, J. B., and Rajagopal, K. R., "Probabilistic Structural Analysis Methods and Applications," *Computers and Structures*, Vol. 30, No. 1/2, pp. 163-170, 1988.
- 3.32 Ditlevsen, O., "Narrow Reliability Bounds for Structural Systems," *Journal of Structural Mechanics*, ASCE, Vol. 112, No. 1, pp. 85-104, 1979.

## 4. NESSUS APPLICATION STUDIES AT ROCKETDYNE

### 4.1 Summary

This section provides a concise summary reference of Rocketdyne's effort in the PSAM contract. To provide the readers with a more complete reference source additional related work performed at Rocketdyne not funded under this contract are also briefly described and referred to. Many of the system loads used in the examples were obtained from the Rocketdyne effort in the Composite Load Spectra Contract (NAS3-24382 and NAS3-26371). Readers interested in obtaining more details of the effort are referred to the cited references. The references are either published papers in the conferences, annual contractor's reports to NASA, contract briefings to the NASA contract monitor, short course notes at NASA Marshall Space Flight Center and in rare instances Rocketdyne Internal reports.

The probabilistic structural analysis technology developed under this contract and applied to a representative set of aero-space components has great relevance to the aerospace industry in today's engine development environment. Increasing emphasis is being placed on low-cost engine development without sacrificing the reliability of the engine systems. The old method of test, fail, fix approach though effective is expensive relative to the available budget. Thus a comprehensive set of tools that provide a handle in identifying the sources of the unreliability, their relative importance, and designing the hardware for high reliability in the presence of controllable and uncontrollable variability and uncertainty will go a long way in developing a low-cost engine development program. The same tools also provide the technology for managing the risk during the production phase of the program. The PSAM developed technology has provided the tools and showed an approach to achieving a low cost engine development program.

This report provides a summary overview of the PSAM technology application to many SSME and other aerospace components. It includes turbine blades, main combustion chamber injector element, high pressure pump discharge duct, main combustion chamber liners, coolant liner for a hot gas manifold, welds, castings and other hardware.

The applications covered a variety of structural and non-structural mechanics areas. In the area of structural analysis, they included static linear and nonlinear analysis, modal analysis, dynamic response analysis due to harmonic and random excitation, static buckling analysis and fatigue damage evaluations due to low cycle and high cycle fatigue. Nonstructural analysis areas included thermal analysis, systems analysis that included linking geometry stack up and tolerance computation programs, fluid and rotor dynamics programs and casting porosity prediction methodology. Many of the applications included detailed finite element models and deterministic analysis techniques of the same fidelity as were used in the current analysis techniques, only that a probabilistic analysis layer was introduced on top of the deterministic

methods. Further, a variety of numerical models were exercised by the probabilistic analysis driver such as a combination of analytical models obtained through numerical and/or experimental techniques, finite element and other closed form solutions as necessary.

Though not contractually required many applications included interfacing with Composite Load Spectra computer code (developed under separate NASA Contract NAS3-24382 and NAS3-26371) to provide engine loads necessary for the probabilistic structural analysis.

Application of probabilistic methods demonstrated in the examples included Monte Carlo Simulation, Latin Hypercube, Mean Value First Order, and Advanced Mean Value First Order though not all were exercised in each example.

#### **4.2 Major Objectives of Rocketdyne's Effort**

Rocketdyne's tasks included providing expertise on the hardware environment and on the analysis and design issues of aerospace propulsion system hardware to the PSAM contractor team. Rocketdyne was also responsible to exercise, demonstrate and validate the applicability of PSAM-NESSUS and CLS-NESSUS combination codes to the analysis of aerospace hardware. Rocketdyne's responsibility included assimilation of the technology and demonstrate the methods to better the future designs of aerospace hardware. The following summary and the cited references demonstrate that the above objectives have been met.

#### **4.3 Space Shuttle Main Engine (SSME) Hardware Analysis**

##### **4.3.1 High Pressure Fuel Turbo Pump (HPFTP) Second Stage Turbine Blade-static Linear Analysis**

This was the first application of the CLS-NESSUS package of computer codes to the SSME Hardware [Reference 4.1]. The objective of the analysis was to predict the variations in the effective stress at critical and other nodal locations of the structural finite element model. The salient features of the analysis included:

- **Variables**

- Consideration of geometry variations from blade to blade due to manufacturing machining considerations.
- The variations in elastic modulus, shear modulus and Poisson's ratio from material stock to stock.
- The variation in material axis orientations (Nickel based super alloy) from blade to blade due to variations in the casting process.
- Considerations of global engine system variables such as mixture ratio, fuel and oxidizer inlet pressure and temperature.

- Local pump to pump variables such as pump efficiency and head coefficient.
- Other local model uncertainty variables such as coolant and hot gas seal leakage.

- **Analysis Features**

- An approach to effectively link CLS with NESSUS/FEM to predict the variations in thermal field, pressure field and speed due to the variations in the engine operating variables. The variations of pressure, thermal and speed were captured in a correlated manner to accurately predict the variations in stresses at different engine operating points.
- Probabilistic analysis using Mean Value First Order Method (MVFO).

- **Results**

- Mapping of mean, coefficient of variation and probabilistic sensitivity factors over the entire finite element model.
- Relative ranking of variables based on their effect on probability values for the effective stress at critical locations.
- Mean Value First Order solutions can be updated to obtain more accurate probabilistic analysis solutions but only for select number of critical points due to the computational cost.

#### **4.3.2 High Pressure Fuel Turbo Pump (HPFTP) Second Stage Turbine Blade - A Proposed Design - Modal Analysis**

This application was concerned with the probabilistic modal analysis of a turbine blade [Reference 4.2]. The natural frequencies of the turbine blade are closely monitored and controlled to avoid interference with engine orders. The reliability of a turbine blade is critically affected by its natural frequency characteristics. Design considerations include natural frequency margins for interference with engine order. The objectives of the analysis were:

- Based on past experience on primitive material and manufacturing variability can the expected variability in blade frequencies be predicted?
- If so, can the blades be designed consistent with the variations such that interference with engine orders can be specified to a prescribed reliability limit?

- **Variables**

- Material orientation angles for the Nickel based super alloy.
- Material elastic properties elastic modulus, Poisson's ratio and shear modulus.

- Geometric variations due to manufacturing process.
- Variations in mass density.

- **Analysis Features**

- Eigenvalue analysis of perturbed structures through re-analysis using subspace iteration and using a more efficient Eigen extraction procedure.
- Mean value First Order and Advanced Mean Value First Order.

- **Results**

- New techniques [Reference 4.3] were developed by SWRI to handle response functions which are non-monotonic using Advanced Man Value First Order Method.
- AMV+ methods were able to predict highly skewed response distributions and relative ranking of the variables.
- The low order statistics obtained from the analysis (mean, standard deviation) were consistent with practical experience.

#### **4.3.3 High Pressure Oxidizer Turbopump (HPOTP) Discharge Duct**

This application was a demonstration of probabilistic analysis techniques in the area of linear dynamic analysis [Reference 4.4]. The analysis involved random vibration and harmonic analysis of a duct subjected to a multibase excitation. While random vibration itself is probabilistic in nature, this analysis considered the uncertainties in the parameters of the stochastic excitation and the uncertainties in the system parameters in a probabilistic analysis format. A more detailed discussion of the many features of the dynamic analysis and the uncertainties involved as applied to the analysis and design of aerospace hardware is discussed in a review paper [Reference 4.5].

- **Variables**

- Variations in mean square power as observed experimentally from SSME zonal Power Spectral Densities (P.S.D.'s).
- Variations in pump speed and the related harmonic frequencies from SSME test data.
- Variations in harmonic excitation amplitudes.
- Uncertainty in damping.
- 38 independent random variables were considered in the analysis.

- **Analysis Features**

The analysis features a state of the art dynamic response analysis due to a simultaneous multibase and multiaxis dynamic random and harmonic excitation. This should be contrasted with industry standard one axis at a time vibration analysis methodology.

- Simultaneous Multibase and Multiaxis random excitation either fully correlated or uncorrelated using the big mass approach.
- Multi frequency harmonic excitation correlated to pump speed.
- Efficient resolution techniques for perturbations that involve dynamic response computation skipping repetitive modal analysis, if necessary.
- Superposition of several spectral cases to obtain a single composite dynamic response.

- **Results**

- MVFO and AMV solutions to obtain the probabilistic dynamic response (cross-sectional moments) at critical locations. The results included cumulative distribution function, mean and standard deviations.
- Sensitivity information for the full range of the response.

#### **4.3.4 SSME Main Injector Row 13 and Row 12 Injector Elements (LOX Post) - Static Non-linear and Fatigue Damage Analysis**

This analysis propagated the variability from system loads, material property, and other model uncertainties to the end injector reliability against fatigue failure. Two separate regions of the LOX post (injector) were considered in two separate analysis. The first analysis [Reference 4.6] considered the effective strain range at the critical inertia weld location of the Row 13 Lox Post.

##### **First Analysis**

- **Variables**

- Engine system variables: hot gas temperature, coolant temperature, coolant flow rate and mixture ratio.
- Thermal modeling uncertainties: heat-shield gap factor, hot gas film coefficient and coolant film coefficient.
- Material property: variable yield stress.

- **Analysis Features**

- A scaling technique for scaling the temperature profile thorough out the LOX post given the surface temperature changes.
- Incremental static nonlinear analysis.
- Incorporation of procedures to model accurately the time dependent variations of the thermal profile using user defined perturbation (UPERT) routines in NESSUS/FEM.

- **Results**

- Probability distribution of the effective strain range at the critical location that contributes to the Low Cycle Fatigue damage.

### **Second Analysis**

The second analysis of the LOX Post [Reference 4.7] put together a comprehensive set of variables and linked them to fatigue damage to compute the reliability of the component at the critical thread location against fatigue failure.

- **Variables**

- A comprehensive set of sixteen engine system variables such MCC injector resistance, oxidizer inlet temperature and pressure etc.
- Damping uncertainty.
- Correlation length uncertainty for the random pressure excitation.
- Mean square power uncertainty for the random pressure.
- Fatigue resistance variability.

- **Analysis features**

- Random pressure excitation analysis with linear distance dependent correlation model.
- An efficient computation scheme through explicit variable definition and user defined Z-function approach for R.M.S. stress computation due to power change but no shape change of the P.S.D.
- Superposition of multi-base spectral analysis results and pressure excitation results.
- A detailed fatigue damage calculation module for low cycle fatigue and high cycle fatigue under spectrum loading.

- A response surface model to predict the static nonlinear response at the key strain reversal points of the hysteresis loop computed at the critical thread location.
  - A finite element model to compute the dynamic response.
  - A key feature of the analysis is the approach to capture the correlation effect among the local variables through the primitive variables definition and the system model.
  - Probabilistic analysis using MVFO, AMV, Monte Carlo and Latin Hypercube methods.
- **Results**
    - Probability fatigue damage prediction at the critical thread location and the corresponding sensitivity factors.
    - Comparison of results between MVFO, AMV, Monte Carlo and Latin Hypercube methods.

#### **4.3.5 SSME Transfer Tube Liner (Two-duct) - Static Linear Buckling Analysis**

This application dealt with predicting buckling strength of a doubly curved hot gas manifold liner shell (SSME two duct transfer tube liner) probabilistically [Reference 4.8]. It used a global response surface approach to predict the buckling strength of a doubly curved shell due to the external pressure.

- **Variables**
  - The thickness of the shell in the five zones which were treated independent.
- **Analysis Features**
  - A subspace iteration procedure to compute the static linear buckling pressure.
  - A response surface approach to construct the response surface considering the five zonal thickness with mixed terms only in the critical region.
- **Results**
  - Cumulative distribution function and sensitivity factors for the buckling strength using Monte Carlo and AMV methods.

#### **4.4 Related Technology Applications**

The technology developed by this contract was demonstrated to be applicable for the analysis and design of many other aerospace components through independent case studies by

Rocketdyne. Though not funded through this contract, a concise summary of the studies is given here to provide the more complete overview of the breadth of this technology application at Rocketdyne.

#### **4.4.1 Structural Mechanics Applications**

##### ***4.4.1.1 SSME High Pressure Oxidizer Turbopump (HPOTP) Bearing Cartridge - Probabilistic Fatigue Damage Computation Due to Transient Resonance***

The bearing cartridge provides support to the bearing cage acting as a spring and is used to isolate the bearing from pump loads at the turbine end. The cartridge had a potential resonance problem at 4N (4 times the speed of the shaft) at 100% power level. Since the engine does not operate at 100% power level for any significant amount of time, the resonance occurred only during the ramping of the power level of the engine. Since the speed of the shaft is a dependent variable and only the power level is controlled, there was a potential and an uncertainty that the pump might run at resonance occurring speeds even when the power level is not 100 % depending upon a variety of factors such as engine inlet conditions. The task was to assess the risk due fatigue failure considering several uncertainty factors that affected engine operating conditions and consequently the fatigue damage. A brief summary of this analysis is given in [Reference 4.9]. This cited reference is a good source wherein complete detailed annotated computer decks for many of the problems cited in this report can be found.

- **Variables**

- Natural frequency of the cartridge.
- Speed at 100% Power Level.
- Jump in speed at the instant of solid rocket motor separation.
- Speed change/unit time at steady state power level-Phase I.
- Speed change slice to slice random variation-Phase II.
- Damping.

- **Analysis Features**

- The strain prediction was based on an empirical model using measured strain gage responses.
- Bearing cartridge natural frequencies statistics were from experimental bench test data.

- Analysis considered the warming of the cartridge with engine operation, relaxing of the preload as a function of time from the start with a corresponding rate change of fatigue damage accumulation due to mean stress variation.
- Cartridge accrued damage in a very narrow speed band due to the lightly damped system (<1.0%).
- The analysis kept major uncertainty factors as random while keeping numerous other variables at their extreme (worst) condition.
- The speed variation of the pump shaft was directly modeled from flight data and the Composite Load Spectra engine model was used to predict the speed.
- Monte Carlo Simulation technique was applied.

- **Results**

- Computation of damage at 0.9, 0.99, and 0.999 probability levels and average expected damage per flight.
- Sensitivity analysis of damage at several mean damping ratio values.

#### **4.4.1.2 SSME Main Combustion Chamber (MCC) Liner-Non-Hot Spot-Probabilistic Low Cycle Fatigue (LCF) Damage Assessment**

The SSME is protected by an actively cooled liner system [References 4.9, 4.10]. The liner is made of a copper-zirconium alloy NARLOY-Z. The support system for the liner includes a structural jacket made of Inconel-718. The cold hydrogen running through the liner channels carry the heat and protect the structural jacket from unacceptable high temperatures. The combustion chamber side of the liner itself has non-uniform temperature profile both axially and circumferentially. All liners are subjected to varying degree of localized hot spots due to the non-uniformity in combustion and due to injector orientation angle variations. Since the liner is subjected to thermal cycles during each start-stop cycle they are designed to withstand the low cycle fatigue damage during their life cycle. This study probabilistically quantified the LCF damage considering the many uncertainties that contribute to the LCF life of the liner.

- **Variables**

- Engine system variables such as flow meter measurement error, MCC pressure measurement error, nozzle area, efficiency multipliers.
- Local modeling uncertainties such as coolant curvature enhancement, hot gas and coolant film coefficient factor.

- Material uncertainties such as fatigue resistance, thermal conductivity.
- Geometric variables such as liner hot gas wall thickness.

- **Analysis Features**

- Use of the Composite Load Spectra Code to model the interdependent local variables that affect the temperature field.
- A response surface model for computing the thermal field using local variables such as hot gas and coolant coefficient, hot gas wall thickness, thermal conductivity factor, coolant flow factor and coolant curvature enhancement. Some of the local variables themselves are obtained from CLS interface.
- Several mechanical response surface models for defining the critical points of the hysteresis loop to compute the low cycle fatigue strain range for each duty cycle.
- Linking of external structural analysis solver ABAQUS to NESSUS Probabilistic system to compute the nonlinear structural response.
- Integration of a damage computation module that included Total Strain Range Partitioning technology to compute the LCF damage.
- Probabilistic methods AMV and MVFO were applied on both response surface and direct FEM models to compute the cumulative distribution function of fatigue damage. Additional verification of the methods through Monte Carlo simulation on response surface models.

- **Results**

- Cumulative distribution function of fatigue damage at non-hot spots of the MCC liner.
- Physical and probabilistic sensitivity factors of the variables and their ranking.

The analysis that was described above showed promise and potential to be integrated with advanced health monitoring systems as a spin off technology for potential use as an onboard numerical model. A review of the proposed technique is discussed in detail in this cited reference [4.10] for the MCC liner and for the LOX post in Reference 4.18.

#### ***4.4.1.3 SSME Heat Exchanger Turn Around Vane - Probabilistic High Cycle Fatigue Damage Reliability Assessment***

During the testing of integration of alternate Turbopump Design (ATD) oxidizer pump in to SSME, the heat exchanger turn around vane which is a part of the hot gas

manifold experienced fatigue cracking. The purpose of this task was to analyze the causes of the failure, identify the important random variables that influence this failure mode and evaluate a new design from reoccurrence of this type of failure. This task made important advances in predicting the dynamic stress response due to random pressure loading on the turnaround vane. In particular, a random pressure correlation model was developed that represented the turbulent boundary layer flow. This frequency dependent correlation model was based on experimental air flow data. The model was coded in to NESSUS/FEM computer code and is usable for computing dynamic response due to random pressure for many other components subjected to similar excitations. Some of the details of the study are available in the interim reports as NASA briefings [References 4.11, 4.12]. More complete details of the study will be reported as contractor report to NASA Lewis Research Center (NAS3-26371).

- **Variables**

- Significant independent engine system variables that affect the oxidizer pump turbine exhaust variables such as velocity, mass flow rate and density.
- Geometric variables such as stress concentration, inner and outer vane thickness.
- System parameter variability such as damping and convection velocity ratios.
- Material fatigue resistance variable.

- **Analysis Features.**

- Analysis of air flow data to obtain scaled random pressure P.S.D. and constants for frequency dependent correlation model.
- Dynamic response computation due to random pressure excitation using a frequency dependent correlation model.
- A CLS code interface to NESSUS to obtain engine dependent variables.
- An integrated HCF fatigue damage computation code.
- Analysis results anchored to failure data from ATD experience and no failure data from Rocketdyne HPOTP pump history.

- **Results**

- Probabilistic R.M.S. stress computation at the critical location for baseline and redesign cases along with sensitivity factors.
- Probabilistic fatigue damage computation at critical location for the baseline and redesign cases along with the sensitivity factors.

#### **4.4.1.4 Probabilistic Assessment of Competing MCC Liner Designs**

All the case studies described in the earlier chapter are for the existing designs of the SSME components. However, it was recognized that the real value of this technology is in its application to new designs. The example summarized here [Reference 4.13] was to use the technology to assess different new designs. This application assessed the reliability of three designs for a main combustion chamber liner for the low cycle fatigue failure mode.

- **Variables**

- Geometric Variables thickness and width of the channel.
- Local thermal load variables from engine model.
- Heat transfer modeling uncertainties.
- Local mechanical load variables.
- Damage resistance material variables (ductility, ultimate stress).

- **Analysis Features:**

- An integrated system of in-house codes (i.e., thermal code CHANNEL) and fatigue damage computation code with external codes ANSYS (for mesh generation), ABAQUS (nonlinear structural analysis).
  - Perturbation evaluation through distributed processing.
  - Design tool development to find robust designs.

- **Results**

- Probabilistic LCF damage assessment for three alternate designs.
- Physical and probabilistic sensitivities under normal conditions.
- Comparison with deterministic designs.
- Maximum design condition response.

#### **4.4.1.5 Space Station Battery Weld Analysis - Taguchi and Probabilistic Analysis**

This was the first example application wherein Taguchi concepts for robust design were evaluated along with probabilistic analysis concepts developed under this technology contract [Reference 4.13]. The dispersion of the conventional factor of safety (margin) was evaluated at a weld location considering the variations found in the weld geometry due to manufacturing processes. It was shown that Taguchi type output measures such as signal/noise ratio could be obtained through AMV class probabilistic analysis.

- **Variables**

- Geometric parameters that define the local weld geometry that include thickness of the three pieces that join the weld, offset between the parts, suction curvature, radii at the weld intersections etc.
- Three mean thickness values with corresponding distributions and limits.

- **Analysis Features**

- Quadratic response surface generation using ANSYS finite element code for all components of stress and strain.
- Probabilistic response evaluation for membrane and bending safety factors, proof and operating strain ranges using MVFO, AMV and Monte Carlo.
- Given the limits of the variables, computation of the best and worst configuration using mathematical optimization techniques.

- **Results**

- Response distribution for factor of safety due to membrane plus bending of the axial component.
- Probabilistic and physical sensitivity factors.
- Comparison of best and worst case designs obtained through optimization with low and high probability factor of safety occurrence of designs.
- Comparison of the conventional deterministic two and three sigma (input parameters) design with the true output sigma values.
- Computation of signal to noise ratios and display of the same computed through probabilistic analysis.

## **4.4.2 Non-structural Mechanics Applications**

### ***4.4.2.1 Probabilistic Design and Analysis Applied to Thermal Loading Environments and Thermal Responses***

It was recognized early in the application phase that the Probabilistic Concepts are very general and can be applied to any numerical model to ascertain the propagation of the uncertainties through out the system and their effect on the output response. In the arena of rocket propulsion, thermal stresses are major a driver for life and durability of the components. Hence, the first non-structural applications were in the field of thermal analysis.

This paper [Reference 4.14] presents a general overview of the probabilistic thermal analysis methodology developed and applied to aerospace components at Rocketdyne. The details of various levels of modeling from the overall engine system performance model which provides major inter-component loads to local detailed finite difference or finite element models are described. An approximate scaling procedure to obtain detailed thermal field solutions for slight perturbations in the primitive fundamental variables is described. The paper describes the developed interface with SINDA (a thermal analysis code, [Reference 4.15]). The results from a probabilistic thermal analysis of a main combustion chamber liner is presented.

- **Variables**
  - Hot Gas Film Coefficient.
  - Coolant Film Coefficient.
  - Wall Thickness.
  - Thermal Conductivity.
  - Channel Resistance.
  - Curvature Enhancement.
- **Analysis Feature**
  - Thermal Analysis using SINDA and NESSUS/FPI MVFO method.
- **Results**
  - Probabilistic thermal response and the corresponding sensitivity factors.

#### **4.4.2.2 A Probabilistic Casting Porosity Prediction - Methodology**

Investment castings are a low cost manufacturing approach used in aerospace manufacturing for complex shaped parts, provided that casting imperfections like porosity and distortions can be controlled. Traditionally a heuristic approach based on experience is used to locate risers to manage the flow, filling and solidification of casting. This is followed by a series of cut and try or design of experiments approach of changing critical parameters to eliminate casting defects and improve material properties.

The work cited here [Reference 4.16] is part of the contract for the Development of Life Prediction Capabilities for Liquid Propellant Rocket Engines, Manufacturing Requirements from a PDA Approach (NAS3-25884, Task Order-6). The ongoing work under this task is using Niyama's criterion to predict the porosity formation for a SSME component.

- **Variables**

- Pour temperature.
- Pre-Heat temperature.
- Thickness of the part.
- Molten material conductivity uncertainty.
- Gap conductance uncertainty between ceramic mold and casting.

- **Analysis Features**

- Thermal analysis using SINDA.
- A Niyama's criterion computation post processor.
- NESSUS/FPI AMV and Monte Carlo Solutions.

- **Results**

- Cumulative Distribution Function for Niyama's criterion and the corresponding sensitivity factors at the critical location.

#### **4.4.2.3 Turbopump Preliminary Design Using Probabilistic Analysis and Robust Design Concepts**

This task conducted using Rocketdyne internal funds [Reference 4.17] applied the Probabilistic Analysis and Robust Design concepts to a preliminary turbopump design. It is an attempt to perform a multi-disciplinary optimization of the design considering the uncertainties even at the preliminary design level. It is a systems

approach to provide probabilistic responses and the identification of the important random variables and their sensitivity factors for each discipline and bring forth the interrelationships of the various performance measures. The analysis highlights include integration of manufacturing capability data base, geometry stack up model or (VSA), a turbopump performance model and a rotor dynamics model.

- **Variables**

- A total of one hundred and six variables were considered, but not all are significant.
- Some physically significant variables include pump discharge mass flow rate, pump inlet temperatures and pressures, pump discharge pressure and turbine discharge pressure.
- Some probabilistically significant variables include rotor geometric characteristics such as rotor, impeller and inducer dimensional tolerances.
- Rotordynamics inducer and impeller unbalance characteristics.

- **Analysis features**

- Linking a response surface based VSA model.
- Linking of Turbopump performance and rotordynamics model.
- Monte Carlo simulation and AMV solutions using NESSUS/FPI.

- **Results**

- Probabilistic pump responses such as speed, suction performance margin, balance piston forces, etc.
- Deflection of mechanical elements such as inducer, wear ring, damper seal.
- Rotor frequencies and critical speed margins.
- Cumulative distribution and sensitivity factors.

#### **4.4.3 General Concept and Review Papers**

During the course of the PSAM and CLS contract Rocketdyne personnel authored several overview papers describing the application of probabilistic analysis methodology to mechanical design. These papers are cited in references [References 18-22].

#### **4.5 Concluding Remarks**

Rocketdyne has successfully applied the technology to the analysis of aerospace components. It has

- demonstrated through examples the possible depth and breadth of the application of probabilistic analysis methods to structural analysis.
- extended the technology to other disciplines as well such as thermal analysis and systems analysis.
- demonstrated the linkage of the numerical engine propulsion load models (CLS) to NESSUS family of computer codes.
- applied the technology to case studies of new designs.

#### **References**

- 4.1 Newell, J. F., Rajagopal, K. R., and Ho, H., "Probabilistic Structural Analysis of Space Propulsion System Turbine Blades," AIAA 30th Structures, Structural Dynamics and Materials Conference, Mobile, Alabama, April 3-5, 1989.
- 4.2 NASA Lewis Research Center, "Probabilistic Structural Analysis Methods for Select Space Propulsion System Components, Contractor's Fourth Annual Report," NAS3-24389, Cleveland, Ohio, 1988.
- 4.3 Wu, Y.-T. Wu, Millwater, H. R., Cruse, T. A., "Advanced Probabilistic Structural Analysis Method for Implicit Performance Functions," *AIAA Journal*, Vol. 28, No. 9, September 1990.
- 4.4 DebChaudhury, A., Rajagopal, K. R., Ho, H., and Newell, J. F., "A Probabilistic Approach to the Dynamic Analysis of Ducts Subjected to Multibase Harmonic and Random Excitation," AIAA 31st Structures, Structural Dynamics and Materials Conference, Long Beach, California, April 1990.
- 4.5 Rajagopal, K. R., DebChaudhury, A., and Newell, J. F., "Reliability Analysis of Advanced Space Propulsion Components," ASME Winter Annual Meeting, Anaheim, California, November 10, 1992.
- 4.6 Newell, J. F., Rajagopal, K. R., Ho, H. W., and Cunniff, J. M., "Probabilistic Structural Analysis of Space Propulsion System LOX Post," AIAA 31st Structures, Structural Dynamics and Materials Conference, Long Beach, California, April 1990.
- 4.7 Rajagopal, K. R., Keremese, J., Ho., H., and Orient, G., "A Probabilistic Approach to the Evaluation of Fatigue Damage in a Space Propulsion System Injector Element", AIAA 33rd Structures, Structural Dynamics and Materials Conference, Dallas, Texas, April 1992.

- 4.8 NASA Lewis Research Center, "Probabilistic Structural Analysis Methods for Select Space Propulsion System Components," Contractor's Fifth Annual report, NAS3-24389, Cleveland, Ohio, 1989.
- 4.9 Rocketdyne, "Probabilistic Structural Analysis: A Short Course , Volume I and Volume II," conducted at NASA Marshall Space Flight Center, September-October 1992.
- 4.10 Orient, G. E., Ho, H., Rajagopal, K. R., and McGaw, M. A., "Development of Life Prediction Capabilities for the SSME MCC Liner," *Proceedings, Fourth Annual Health Monitoring Conference for Space Propulsion System*, University of Cincinnati, November 17-18, 1992.
- 4.11 Rocketdyne, "SSME Heat Exchanger Turn Around Vane PSAM/CLS Demonstration Project," Briefing to NASA Contract Monitor Dr. Chamis, April 7, 1994.
- 4.12 Rocketdyne, "SSME Heat Exchanger Turn Around Vane PSAM/CLS Demonstration Project," Rocketdyne Briefing to NASA Contract Monitor Dr. Chamis, March 23, 1995.
- 4.13 Orient, G. E., Rajagopal, K. R., O'Connor, D. L., and Newell, J. F., "Manufacturing Requirements from a Probabilistic Design Analysis Approach to Optimizing for Robust Design," IUTAM Symposium on Probabilistic Structural Mechanics, San Antonio, Texas, June 7-10, 1993.
- 4.14 Newell, J. F., Ho, H. W. Romine, W. D., Depsky, J. S., and Rajagopal. K. R., "Probabilistic Design and Analysis Applied to Thermal Loading Environments and Thermal Responses, 3rd ASME-JSMA Thermal Engineering Joint Conference, Reno, Nevada, Mar 17-22, 1991.
- 4.15 "Systems Improved Numerical Differencing Analyzer and Fluid Integrator (SINDA85/FLUINT), User's Manual," NASA/JSC Contract Number NAS9-17448, 1990.
- 4.16 "Development of Life Prediction Capabilities for Liquid Rocket Propulsion Systems - Task Order-6, Manufacturing Requirements From a PDA Approach," Monthly Progress Reports 1994-1995, NAS3-25884
- 4.17 Orient, G. E., "Probabilistic System Performance Analysis, RAPID (Rocketdyne Advanced Project Integrated Design, Internal Report, 1995.
- 4.18 Rajagopal, K. R., Orient, G., Newell, and McGaw, M., "An Advanced Method for Tracking the Evolution of Fatigue Damage in Reusable Space Propulsion Systems," Third Annual Health Monitoring Conference for Space Propulsion Systems, Cincinnati, Ohio, Nov. 13-14, 1991.

- 4.19 Newell, J. F., and Rajagopal, K. R., "Application of Probabilistic Methods to the Structural Analysis of Liquid Rocket Engine Components," 93-GT-238, International Gas Turbine and Aeroengine Congress and Exposition, Cincinnati, Ohio, May 24-17, 1993.
- 4.20 Newell, J. F., and Rajagopal, K. R., "Integrated Structural Risk-Based Approach for Design and Analysis of Combustion Chamber Liners," AIAA/SAE/ASME/ASEE 28th Joint Propulsion Conference, Nashville, Tennessee, July 6-9, 1992.
- 4.21 Rajagopal, K. R., DebChaudhury, A., and Newell, J. F., "Verification of NESSUS Code on Space Propulsion Components," *Proceedings of ICOSSAR'89*, Vol. III, the 5th International Conference on Structural Safety and Reliability, San Francisco, Aug 7-11, 1989.
- 4.22 O'Hara, K. J. "Liquid Propulsion System Reliability - Design for Reliability," AIAA 25th Joint Propulsion Conference, Monterey, California, July 10-12, 1989.

## **5. NESSUS DOCUMENTATION**

This section outlines the over eight hundred pages of NESSUS documentation. The NESSUS program consists of just over nine hundred subroutines and approximately 140,000 lines of Fortran code. The program is modular in nature and where appropriate the documentation is contained in separate user's manuals for each module. A separate section describes several utilities developed under the NASA program and SwRI internal funding. These utilities have varied functions including NESSUS input file generation and verification, results processing, program modification and file management. The following sections describe the different user's manuals outlining the use and capabilities of the NESSUS program.

### **5.1 Getting Started**

This manual presents several simple example problems to help illustrate the basic ideas and typical approaches one might use for probabilistic analysis using NESSUS. Only some of the features and options of NESSUS are presented in this section but should provide a starting point of how to attack more complex problems. The example problems include several component and system problems.

### **5.2 NESSUS/PFEM User's Manual**

This manual describes the capabilities and use of the NESSUS/PFEM module. PFEM coordinates FPI with the FEM or BEM modules of NESSUS for component risk assessment. PFEM implements the advanced mean value algorithm with iterations (AMV+). PFEM also integrates the structural model (FEM or BEM) with other failure models or analysis capabilities defined in user-written subroutines. With the user subroutines the performance can be more general than the structural response.

### **5.3 NESSUS/FPI User's Manual**

This manual provides detailed instructions on the usage of the NESSUS Fast Probability Integration (FPI) module. The FPI code is a probabilistic analysis tool that implements a variety of methods for probabilistic engineering analysis and design for both component and system problems.

This user's manual gives an overview of the current capabilities of FPI, a description of the input and output files of FPI, the different methods used to describe the g-function, or limit state, and the available solution techniques. A complete summary of the keywords available in FPI is provided along with sample problems.

### **5.4 NESSUS/FPI Theoretical Manual**

This manual provides an overview of the theoretical background of the NESSUS Fast Probability Integration module. The theoretical algorithms and concepts on which FPI are based are discussed in detail.

This manual summarizes the available solution methods in the FPI code listed below.

- First-order reliability method (FORM)
- Second-order reliability method (SORM)
- Advanced first-order method using three-parameter normal (FPI)
- Convolution methods (CONVX, CONVU)
- Standard Monte Carlo method (MONTE)
- Radius-based importance sampling method (ISAMF, ISAMR)
- Adaptive importance sampling methods (AIS1, AIS2)
- Advance mean-based method (MV, AMV, AMV+)
- Fault tree method (FTREE)

Most of the methods are approximate in nature. Based on the FPI concept, these methods usually provide very efficient analysis relative to the standard Monte Carlo method which provides asymptotically exact solution as the number of random sampling approaches infinity.

The selection of methods must consider required computational time, which strongly depends on the complexities of the performance functions (e.g., finite element models versus closed form equations), and solution accuracy, which generally depends on the nonlinearity and smoothness of the performance functions.

The NESSUS/FPI Theoretical Manual provides background information followed by the descriptions of each of the methods. For further details about each method, selected references are provided at the end of each section.

### **5.5 NESSUS/SRA User's Manual**

This manual describes the capabilities and use of the NESSUS/SRA module along with several example problems. The NESSUS System Risk Assessment (SRA) module coordinates the operations between the PFEM and FPI modules to perform probabilistic fault tree analysis. NESSUS/SRA is based on the limit-state definition of the bottom events, as opposed to traditional fault-tree analysis where only a probability of failure is used to define the bottom event. This allows NESSUS to correctly account for dependencies caused by common random variables between the bottom events. Each bottom event can be defined by a finite or boundary element model, closed-form equation, or a user-programmed function.

### **5.6 NESSUS/RISK User's Manual**

This manual describes the capabilities and use of the NESSUS/RISK module along with an example problem. The RISK module computes risk with respect to cost, performance, and a user defined criteria.

### **5.7 NESSUS/SIMFEM User's Manual**

This manual describes the capabilities and use of the NESSUS/SIMFEM module. The SIMulation Finite Element Module (SIMFEM) is a sampling analysis method implemented in

NESSUS. In NESSUS 6.2, the SIMFEM capability is limited to component reliability analysis, i.e., limited to one performance or response function. The performance function sampled is defined through the \*PFEM keywords as defined in NESSUS. The purpose of the SIMFEM capability is to provide a verification capability for the approximate probabilistic methods, such as AMV+. SIMFEM can sample random variables according to the Monte Carlo or Latin Hypercube procedure. Correlated random variables can be treated with SIMFEM. In addition, the framework implemented in SIMFEM can be used for implementing other sampling schemes.

## **5.8 UTILITIES**

This manual describes capabilities and use of several utilities associated with the NESSUS program. A brief overview of each of these utilities is given below.

### **5.8.1 PREFPI**

PREFPI is a preprocessor of the FPI code. Its function is to provide instructions to the user, and interact with the user to generate FPI input. The PREFPI code is developed for using the FPI code as an independent module. The code is designed for the user to begin using FPI without having to know, in detail, the theories behind FPI. The PREFPI code is currently limited to the performance functions that can be defined using the user-defined subroutines RESPON and USERES, or by including the performance function Fortran statements in the FPI input file.

PREFPI generates FPI input files (\*.dat) and "form" files (\*.frm) by asking the user a set of designed questions. All the data input by the user is stored in memory for easy modifications during the interactive PREFPI session. All the user inputs are also stored in the form file that summarizes all the available and selected analysis options. The user can modify the FPI input file (\*.dat) or the form file (\*.frm) by running PREFPI.

The form file (\*.frm) contains a concise description of the available FPI capabilities, data requirements, and user selected options and input data. A form file completely defines a probabilistic analysis problem and solution methods.

### **5.8.2 P/NESSUS**

This manual describes the capabilities and use of the P/NESSUS interface. This interface package provides a data exchange process between NESSUS and PATRAN applications. PATRAN is a general purpose pre- and post-processor for finite element modeling. The P/NESSUS interface has the following capabilities

- 1) Translate a PATRAN finite element model (neutral file) to a NESSUS input deck.
- 2) Translate a NESSUS input deck to a PATRAN finite element model.
- 3) Translate a NESSUS POST file to a PATRAN finite element model.

- 4) Translate a NESSUS perturbation data base to PATRAN results files.

#### **5.8.3 NESSUS/SHELL User's Manual**

The NESSUS/SHELL code is a user interface to assist in the probabilistic problem definition. The special requirements include the probabilistic data set construction, NESSUS/FEM options, random variable distribution types and coefficient of variation parameters, and other features. The interface itself uses the X-Windows standard for graphical interfaces. The approach allows for great user-friendliness, as well as a high degree of portability between DOS, VAX, and Unix platforms. SHELL currently supports NESSUS version 5.0.

#### **5.8.4 DECODE and ENCODE**

These two programs allow porting binary files between two different systems by converting the binary perturbation data base into in ASCII format. DECODE translates the binary perturbation database into ASCII format. ENCODE translates an ASCII perturbation database into binary format. NESSUS uses the binary form of the perturbation database during restart analyses.

#### **5.8.5 CHANGE.PARAMETER**

This manual describes the use of the change.parameter Unix C-shell script for modifying the parameters defining the maximum number of random variables, perturbations, and limit states. The script modifies all NESSUS subroutines containing these parameters.

### **5.9 Programmer's Manual**

This manual describes the use of NESSUS. It describes the files and directory structure of the NESSUS distribution. Also included here are the procedures for increasing the core memory size of NESSUS and how to link in a new user subroutine.

### **5.10 NESSUS/FEM User's Manual**

This manual provides detailed instructions on the usage of the NESSUS finite element code. The NESSUS finite element code employs innovative finite element technology and solution strategies. It is the purpose of this manual to acquaint the user with some of the fundamental notions and basic background material needed to effectively use this code and to provide a detailed summary of capabilities.

The NESSUS finite element code was developed to combine the modeling versatility of a modern finite element code with a clean interface to the remaining modules of the NESSUS software system. It provides a choice of algorithms for the solution of static and dynamic problems, both linear and nonlinear, together with a number of innovative perturbation analysis algorithms to evaluate the sensitivity of the response to small variations in one or more

user-defined random parameters. The code is based on a mixed-iterative finite element formulation derived from a three-field variational principle, with the displacements, strains, and stresses obtained directly at the nodes of the finite element mesh. Whenever possible, input quantities are also defined on a nodal basis. As a result, the NESSUS finite element code may differ, in many respects, from the conventional architecture that has been employed in many of the more commonly used displacement-based finite element codes.

### **5.11 NESSUS/PRE User's Manual**

This manual provides detailed instructions on the usage of the NESSUS random field pre-processor. This program may be used to perform many of the data manipulations needed to express the uncertainties in a random field as a set of *uncorrelated* random variables. The resulting random variables may be input as Level 2 perturbation variables into the NESSUS finite element code, and used as the primitive variables for the fast probability integration program.

### **5.12 NESSUS/LEVEL1 User's Manual**

This manual provides detailed instructions on the usage of the NESSUS Level1 post-processor. The Level1 strategy is based on the simplifying assumption that the uncertainties in the problem can be adequately modeled as a set of *global scalings* of the applied force, stiffness, mass and damping matrices. The Level1 post-processor may sometimes be used to estimate the effects of these uncertainties by performing a series of very simple post-processing operations.

### **5.13 NESSUS/PBEM User's Manual**

This manual provides detailed instructions on the usage and capabilities of the NESSUS/PBEM code. Included in this manual is a theory section and programmers notes including a list of subroutines and a program flow chart. The probabilistic boundary element code has been written as a new, stand-alone BEM code that is fully compatible with the NESSUS/FEM data structures. The focus of the implemented BEM code is on static and vibration analysis with elastic, and thermoelastic capabilities. The boundary collocation of particular solutions has been used for vibration and thermoelastic problems. Collocation can be done at the boundary and interior with equal ease. Limited testing of the code has been done.

### **5.14 NESSUS/SYSTEM and /SYSRSK User's Manual (Alternate System Reliability)**

This manual describes the use of the NESSUS/SYSTEM and NESSUS/SYSRSK modules. The SYSTEM module is an alternate method for computing system reliability using a fault tree analysis. This module computes the upper and lower bounds of the system reliability. The SYSRSK module computes system risk by combining the cost of failure, in terms of replacement, inspection, repair, etc of individual components of the system with their probability of failure. The result is an upper and lower bounds on the cost of operating the system.

## 6. SELECTED PSAM PUBLICATIONS

This section provides a list of selected publications that were either fully or partially supported by the PSAM project. To assist the reader in searching for specific information, a short description of the content is added to the end of each publication. The list, in chronological order, consists of papers or reports published in journals, conference proceedings, and contractor's reports, and of NESSUS code documentation.

1. Burnside, O. H., "Probabilistic Structural Analysis for Space Propulsion System Components," Advances in Aerospace Structural Analysis, O. H. Burnside and C. H. Parr (eds.), ASME, New York, NY, pp. 87-102, 1985. (Overview of PSAM program)
2. Dias, J. B., and Nagtegaal, "Efficient Algorithms for Use in Probabilistic Finite Element Analysis," Advances in Aerospace Structural Analysis, O. H. Burnside and C. H. Parr (eds.), ASME, New York, NY, pp. 37-50, 1985. (Overview of NESSUS/FEM perturbation algorithms)
3. Wu, Y.-T., "Demonstration of a New, Fast Probability Integration Method for Reliability Analysis," Advances in Aerospace Structural Analysis, O. H. Burnside and C. H. Parr (eds.), ASME, New York, NY, pp. 63-73, 1985. (Demonstration of the FPI methods)
4. Wu, Y.-T., and Wirsching, P. H., "New Algorithm for Structural Reliability Estimation," *Journal of Engineering Mechanics*, ASCE, Vol. 113, No. 9, pp. 1319-1336, Sept. 1987. (Describes three-parameter fast-probability-intregation method)
5. Wu, Y.-T., Burnside, O. H., and Dominguez, J., "Efficient Probabilistic Fracture Mechanics Analysis," Dominguez, *Proceedings of the Fourth International Conference on Numerical Methods in Fracture Mechanics*, San Antonio, Texas, March 22-26, 1987. (First publication of AMV method, Most Probable Point Locus concept)
6. Cruse, T. A., Burnside, O. H., Wu, Y.-T., Polch, E. Z., Fink, P. K., Dias, J. B., and Rajagopal, K. R., "Probabilistic Structural Analysis Methods for Select Space Propulsion System Structural Components (PSAM)," *Computers & Structures*, Vol. 29, No. 5, pp. 891-901, 1988. (Application of PSAM to a turbine blade)
7. Wu, Y.-T., and Burnside, O. H., "Validation of the NESSUS Probabilistic Finite Element Analysis Computer Program," *Proceedings of the 29th Structures, Structural Dynamics, and Materials Conference*, Williamsburg, Virginia, Part 3, pp. 1267-1274, Paper No. 88-2372, April 18-20, 1988. (Several probabilistic structural analysis example problems using MV, AMV, and AMV+ algorithms examples problems)
8. Wu, Y.-T., and Burnside, O. H., "Efficient Probabilistic Structural Analysis Using an Advanced Mean Value Method," *Proceedings of the ASCE Specialty Conference on*

*Probabilistic Mechanics and Structural and Geotechnical Safety*, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, pp. 492-495, May 25-27, 1988. (Description of AMV method and how to compute statistical moments from the AMV result)

9. Wu, Y.-T., Burnside, O. H., and Cruse, T. A., "Probabilistic Methods for Structural Response Analysis," *Computational Mechanics of Probability and Reliability Analysis*, Wing Kam Liu and Ted Belytschko (eds.), Elmepress International, pp. 181-196. 1989; Presented at the ASME Symposium on Computational Probabilistic Methods, University of California at Berkeley, California, June 20-22, 1988. (Description of AMV Z and P levels method and how to treat correlated random variables)
10. Newell, J. F., Rajagopal, K. R., and Ho, H., "Probabilistic Structural Analysis of Space Propulsion System Turbine Blades," *Proceedings of the 30th Structures, Structural Dynamics and Materials Conference*, Mobile, Alabama, April 3-5, 1989. (First application of the CLS-NESSUS package of computer codes to the SSME hardware)
11. Millwater, H. R., and Wu, Y.-T., "Structural Reliability Analysis Using a Probabilistic Finite Element Program," *Proceedings of the 30th Structures, Structural Dynamics and Materials Conference*, pp. 8146-1851, 1989. (NESSUS methodology and demonstration)
12. McClung, R. C., Millwater, H. R., Wu, Y.-T., Thacker, B. H., and Burnside, O. H., "An Approximate Methods Approach to Probabilistic Structural Analysis," *Proceedings of the 30th Structures, Structural Dynamics and Materials Conference*, pp. 1840-1845, April 1989. (Probabilistic approximate analysis method)
13. Cruse, T. A., Chamis, C. C., and Millwater, H. R., "An Overview of the NASA (LeRC)—SwRI Probabilistic Structural Analysis (PSAM) Program, *Proceedings of ICOSSAR'89*, Vol. III, the 5th International Conference on Structural Safety and Reliability, San Francisco, pp. 2267-2274, August 7-11, 1989. (Overview of PSAM program)
14. Rajagopal, K. R., DebChaudhury, A., and Newell, F., "Verification of NESSUS Code on Space Propulsion Components," *Proceedings of ICOSSAR'89*, Vol. III, the 5th International Conference on Structural Safety and Reliability, San Francisco, pp. 2299-2306, August 7-11, 1989. (Overview of PSAM verification study)
15. Harren, Stephen V., "Probabilistic Analysis of Structures Composed of Path-Dependent Materials," *Proceedings of ICOSSAR'89*, Vol. III, the 5th International Conference on Structural Safety and Reliability, San Francisco, pp. 2291-2298, August 7-11, 1989. (Describes a time-dependent reliability method for path dependent materials)
16. Wu, Y.-T., and Wirsching, P. H., "Advanced Reliability Methods for Probabilistic Structural Analysis," *Proceedings of ICOSSAR '89*, 5th International Conference on Structural Safety and Reliability, San Francisco, California, pp. 2275-2282, August 8-11, 1989. (Summary of AMV method and how to treat correlated random variables)

17. Millwater, H. R., Wu, Y.-T., Dias, J. B., McClung, R. C., Raveendra, S. T., and Thacker B. H., "The NESSUS Software System for Probabilistic Structural Analysis," *Proceedings of ICOSSAR '89*, 5th International Conference on Structural Safety and Reliability, San Francisco, California, pp. 2283-2290, August 8-11, 1989. (Overview of NESSUS program)
18. Cruse, T. A., Unruh, J. F., Wu, Y.-T., Harren, and S. V., "Probabilistic Structural Analysis for Advanced Space Propulsion System," *Journal of Engineering for Gas Turbine and Power*, ASME, April 1990. (PSAM methodology demonstration problems)
19. Wu, Y.-T., and Burnside, O. H., "Computational Methods for Probability of Instability Calculations," *Proceedings of the 31st Structural Dynamics and Materials Conference*, pp. 1081-1091, April 1990. (Application of importance sampling to probabilistic rotor instability analysis)
20. Millwater, H. R., Wu, Y.-T., and Fossum, A. F., "Probabilistic Analysis of a Materially Nonlinear Structure," *Proceedings of the 31st Structural Dynamics and Materials Conference*, pp. 1048-1053, April 1990. (Demonstration of AMV method to material nonlinear structure)
21. Thacker, B. H., McClung, R. C., and Millwater, H. R., "Application of the Probabilistic Approximate Analysis Method to a Turbopump Blade Analysis," *Proceedings of the 31st Structures, Structural Dynamics and Materials Conference*, Long Beach, California, pp. 1039-1047, 1990. (Describes an approximate probabilistic method for handling complex deterministic models)
22. DebChaudhury, A., Rajagopal, K. R. Ho, H., and Newell, J. F., "A Probabilistic Approach to the Dynamic Analysis of Ducts Subjected to Multibase Harmonic and Random Excitation," *Proceedings of the 31st Structures, Structural Dynamics and Materials Conference*, Long Beach, California, April 1990. (Demonstration of probabilistic methods in the area of linear dynamic analysis)
23. Newell, J. F., Rajagopal, K. R., Ho, H. W., and Cunniff, J. M., "Probabilistic Structural Analysis of Space Propulsion System LOX Post," *Proceedings of the 31st Structures, Structural Dynamics and Materials Conference*, Long Beach, CA, April 1990. (Application of NESSUS to SSME injector reliability against fatigue failure)
24. Wu, Y.-T., Millwater, H. R., and Cruse, T. A., "An Advanced Probabilistic Structural Analysis Method for Implicit Performance Functions," *Proceedings of the 30th Structures, Structural Dynamics and Materials Conference*, Mobile, Alabama, April 3-5, 1989, Paper No. AIAA 89-1371; AIAA Journal, Vol. 28, No. 9, pp. 1840-1845, September 1990. (AMV method for nonmonotonic performance functions)

25. Thacker, B. H., Millwater, H. R., and Harren, S. V., "Computational Methods for Structural Load and Resistance Modeling," *Proceedings of the 32nd Structures, Structural Dynamics and Materials Conference*, Baltimore, Maryland, Paper No. AIAA-91-0918, April 7-10, 1991. (Examples combining user-written subroutine UZFUNC and structural analysis with AMV+)
26. Tornq, T. Y., and Wu, Y.-T., "Development of Probabilistic Analysis Methodology for Structural Reliability Estimation," *Proceedings of the 32nd Structures, Structural Dynamics and Materials Conference*, Baltimore, Maryland, April 7-10, 1991. (Description of NESSUS fast convolution method)
27. Millwater, H. R., Harren, S. V., and Thacker, B. H., "Probabilistic Analysis of Structures Involving Random Stress-Strain Behavior," *Proceedings of the 32nd Structures, Structural Dynamics and Materials Conference*, Baltimore, Maryland, Paper No. AIAA-91-0919, April 7-10, 1991. (Describes how to consider a random stress-strain curve using NESSUS)
28. Millwater, H., Thacker, B., Wu, Y.-T., and Cruse, T. A., "Structural Reliability Methods — Code Development Status," *Proceedings of the Structural Integrity and Durability of Resusable Space Propulsion Systems*, NASA Lewis Research Center, Cleveland, OH, No. 10064, pp. 71-75, May 14-15, 1991. (NESSUS 5.0 Summary)
29. Rajagopal, K. R., Newell, J. F., and Ho, H., "Probabilistic Evaluation of SSME Structural Components," *Proceedings of the Structural Integrity and Durability of Resusable Space Propulsion Systems*, NASA Lewis Research Center, Cleveland, OH, No. 10064, pp. 85-95, May 14-15, 1991. (Turbine blade static and modal analysis)
30. Cruse, T. A., and Mahadevan, S., "System Reliability and Risk Assessment Task Goals & Status," *Proceedings of the Structural Integrity and Durability of Resusable Space Propulsion Systems*, NASA Lewis Research Center, Cleveland, OH, No. 10064, pp. 111-120, May 14-15, 1991. (Alternate system reliability and risk assessment)
31. Wu, Y.-T., "Probabilistic Structural Analysis Algorithm Development for Computational Efficiency," *Proceedings, Structural Integrity and Durability of Resusable Space Propulsion Systems*, NASA Lewis Research Center, Cleveland, OH, No. 10064, pp. 121-130, May 14-15, 1991. (Review of NESSUS probabilistic analysis algorithms)
32. Huang, Q., and Cruse, T. A., "Probabilistic Boundary Element Structural Analysis," *Advanced Earth-to-Orbit Propulsion Technology 1992*, NASA Conference Publication No. 3174, Vol. II, pp. 522-531, 1992. (Describes NESSUS/PBEM methodology)
33. Millwater, H., and Wu, Y.-T., "Structural Reliability Assessment (SRA) Capability in NESSUS, Southwest Research Institute, *Advanced Earth-to-Orbit Propulsion Technology 1992*, NASA Conference Publication No. 3174, Vol. II, pp. 511-521, 1992. (Describes NESSUS risk assessment capability)

34. Wu, Y.-T., Tornq, T. Y., and Burnside, O. H., "Probabilistic Rotor Instability Analysis," Southwest Research Institute, Advanced Earth-to-Orbit Propulsion Technology 1992, NASA Conference Publication No. 3174, Vol. I, pp. 508-521, 1992.
35. Mahadevan, S., and Cruse, T. A., "An Advanced First-Order Method for System Reliability," *Proceedings of the Sixth ASCE Joint Specialty Conference on Probabilistic Mechanics and Structural and Geotechnical Reliability*, Denver, CO, American Society of Civil Engineers, New York, pp. 487-490, 1992. (Describes an alternate system reliability analysis method)
36. Riha, D. S., Millwater, H. R., and Thacker, B. H., "Probabilistic Structural Analysis using a General Purpose Finite Element Program," Finite Elements in Analysis and Design, Vol. 11, pp. 201-211, 1992. (Describes using MV and AMV algorithms with MSC/NASTRAN, examples include displacement of a cantilever beam and buckling of a composite plate)
37. Rajagopal, K. R., Keremes, J., Ho, H., and Orient, G., "A Probabilistic Approach to the Evaluation of Fatigue Damage in a Space Propulsion System Injector Element," *Proceedings of the 33rd Structures, Structural Dynamics and Materials Conference*, Dallas, Texas, April 1992. (Application of NESSUS to SSME reliability against fatigue failure)
38. Tornq, T. Y., and Thacker, B. H., "Confidence Bounds Assessment for Probabilistic Structural Reliability Analysis," *Proceedings of the 33rd Structures, Structural Dynamics, and Materials Conference*, Paper No. AIAA-92-2409, Dallas, TX, April 13-15, 1992. (A method for constructing confidence bounds)
39. Thacker, B. H., and Wu, Y.-T., "A New Response Surface Approach for Structural Reliability Analysis," *Proceedings of the 33rd Structures, Structural Dynamics and Materials Conference*, Dallas, Texas, April 13-15, 1992. (Describes an MPP based response surface useful for combining correlated stress and strength variables)
40. Millwater, H., Wu, Y.-T., Tornq, T., Thacker, B., Riha, D., and Leung, R., "Recent Developments of the NESSUS Probabilistic Structural Analysis Computer Program," *Proceedings of the 33rd Structures, Structural Dynamics and Materials Conference*, Dallas, Texas, April 13-15, 1992. (Describes overall structure, modules, and capabilities of NESSUS code, includes fracture mechanics example)
41. Tornq, T. Y., Wu, Y.-T., and Millwater, R., "Structural System Reliability Calculations Using a Probabilistic Fault Tree Analysis Method," *Proceedings of the 33rd Structures, Structural Dynamics and Materials Conference*, Dallas, Texas, April 13-15, 1992. (Describes fault tree system reliability method)
42. Wu, Y.-T., Tornq, T. Y., Burnside, O. H., and Rheinfurth, M. H., "Probabilistic Rotordynamics Analysis Using an Adaptive Importance Sampling Method," *Proceedings of*

*the 1992 ASCE Speciality Conference on Probabilistic Mechanics and Structural Geotechnical Reliability*, Denver, CO, July 1992. (Application of adaptive importance sampling to rotordynamics)

43. Millwater, H. R., and Wu, Y.-T., "Structural Reliability Assessment Capability in NESSUS," *Proceedings of the 1992 JPC AIAA/SAAE/ASME/ASEE 28th Joint Propulsion Conference*, Nashville, TN, July 6-8, 1992. (NESSUS overview)
44. Newell, J. R., and Rajagopal, K. R., "Integrated Structural Risk-Based Approach for Design and Analysis of Combustion Chamber Liners," *Proceedings of the 28th Joint Propulsion Conference*, Nashville, Tennessee, July 6-9, 1992. (Overview of PSAM application to SSME combustion chamber liners)
45. Thacker, B. H., Wu, Y.-T., Millwater, H. R., Tornq, T. Y., Tornq, T. Y., "Recent Advances in Computational Structural Reliability Analysis Methods," *Proceedings, Workshop on Comp. Methods for Failure Analysis and Life Predictions*, Langley Research Center, Hampton, VA, NASA CP-3230, October 13-16, 1992. (State-of-the-art overview and future applications for NESSUS)
46. Wu, Y.-T., "An Adaptive Importance Sampling Method for Structural System Reliability Analysis," *Reliability Technology*, Vol. AD-28, American Society of Mechanical Engineers, New York, pp. 217-231, 1992. (Adaptive importance sampling methodology)
47. Mahadevan, S., Cruse, T. A., Huang, Q., and Mehta, S., "Structural Re-analysis for System Reliability Computation," *Reliability Technology*, Vol. AD-28, American Society of Mechanical Engineers, New York, pp. 169-187, 1992. (Describes a re-analysis procedure for addressing system reliability and progressive damage)
48. Rajagopal, K. R., DebChaudhury, A., and Newell, J. F., "Reliability Analysis of Advanced Space Propulsion Components," *Reliability Technology*, Vol. AD-28, American Society of Mechanical Engineers, New York, pp. 159-198, 1992. (Application of NESSUS to dynamic analysis of aerospace hardware)
49. Orient, G. E., Ho, H., Rajagopal, K. R., and McGaw, M. A., "Development of Life Prediction Capabilities for the SSME MCC Liner," *Proceedings, Fourth Annual Health Monitoring Conference for Space Propulsion Systems*, University of Cincinnati, November 17-18, 1992. (Application of NESSUS to SSME main combustion chamber)
50. Mehta, S. T. Cruse, and Mahadevan, S., "Confidence Bounds on Structural Reliability," *Proceedings of the 34th SDM Structures, Structural Dynamics, and Materials Conference*, La Jolla, CA, April 19-21, 1993. (Describes how to compute confidence bounds)

51. Millwater, H. R., and Wu, Y.-T., "Global/Local Methods for Probabilistic Structural Analysis," *Proceedings of the 34th SDM Structures, Structural Dynamics, and Materials Conference*, La Jolla, CA, April 19-21, 1993. (Describes how to use a coarse and fine deterministic model together for accurate efficient analysis)
52. Mahadevan, S., and Chamis, C., "Structural System Reliability Under Multiple Failure Modes," *Proceedings of the 34th SDM Structures, Structural Dynamics, and Materials Conference*, La Jolla, CA, April 19-21, 1993. (Describes a system reliability analysis method)
53. Wu, Y.-T., "Computational Methods for Efficient Structural Reliability and Reliability Sensitivity Analysis," *Proceedings of the 34th SDM Structures, Structural Dynamics, and Materials Conference*, La Jolla, CA, and AIAA Journal, Vol. 32, Number 8, pp. 1717-1723, April 19-21, 1993. (Adaptive importance sampling based system sensitivity factors)
54. Newell, J. F., and Rajagopal, K. R., "Application of Probabilistic Methods to the Structural Analysis of Liquid Rocket Engine Components," 38th International Gas Turbine and Aeroengine Congress and Exposition, Cincinnati, Ohio, No. 93-GT-238, May 24-27, 1993. (Overview of PSAM application to engine components)
55. Mahadevan, S., Mehta, S., Tryon, R., and Cruse, T., "System Reliability Design Analysis of Propulsion Structures," 38th International Gas Turbine and Aeroengine Congress and Exposition, Cincinnati, OH, May 24-27, 1993. (Describes the application of PSAM to a rotor involving multiple failure modes)
56. Millwater, H. R., and Wu, Y.-T., "Computational Simulation of Structural Reliability Analysis of a Turbine Blade, 38th International Gas Turbine and Aeroengine Congress and Exposition, Cincinnati, Ohio May 24-27, 1993. (Detailed analysis of the system reliability of a turbine blade subject to creep, vibration and stress failure modes)
57. Orient, G. E., Rajagopal, K. R., O'Connor, D. L., and Newell, J. F., "Manufacturing Requirements from a Probabilistic Design Analysis Approach to Optimizing for Robust Design," *Proceedings of the IUTAM Symposium on Probabilistic Structural Mechanics*, San Antonio, Texas, June 7-10, 1993. (Application of PSAM to probabilistic design)
58. Millwater, H. R., Wu, Y.-T., and Cardinal, J. W., "Probabilistic Structural Analysis of Fatigue and Fracture," *Proceedings of the 35th SDM Structures, Structural Dynamics, and Materials Conference*, Hilton Head, SC, April 18-20, 1994. (Advanced fracture mechanics algorithm integrated with finite element analysis using NESSUS)
59. Cruse, T. A., Mahadevan, S., Huang, Q., and Mehta, S., "Mechanical System Reliability and Risk Assessment," *AIAA Journal*, Vol. 32, No. 11, November 1994. (A methodology for system reliability analysis considering accumulated damage)

60. Millwater, H. R., Wu, Y.-T., Cardinal, J. W., and Chell, G. G., "Application of Advanced Probabilistic Fracture Mechanics to Life Evaluation of Turbine Rotor Blade Attachments," Presented at the 1995 IGTI 40th International Gas Turbine Conference, Houston, June 5-8, 1995. (Inspection and repair effects on probability of failure due to fracture)

## **7. SUMMARY AND RECOMMENDATIONS**

The objective of PSAM project was to develop a new technology capability for the probabilistic structural analysis of engine components for current and future space propulsion systems. The PSAM effort consisted of several major technical thrusts: probabilistic finite element methods, probabilistic approximate analysis methods, probabilistic advanced analysis methods, component risk assessment, and system risk assessment.

PSAM has successfully integrated computer-sensitive structural analysis methods (finite elements, boundary elements) with new reliability methods to simulate uncertainties in loads, material, geometries, and boundary conditions. There are many new technologies developed by the PSAM project as summarized in Sections 2 and 3. The application of NESSUS to SSME and other engine components by Rocketdyne is summarized in Section 4. A summary of the comprehensive NESSUS documentation is presented in Section 5. Section 6 lists sixty selected PSAM-related publications.

The capability of NESSUS includes predicting structural reliability, determining dominant variables affecting reliability, performing probabilistic sensitivity to identify data sensitivity/needs, assessing the structural risk in terms of cost, and determining optimum inspection intervals. The analyses can be used to provide information for retirement for cause decisions, certify the structure subject to reliability requirements, and support the definition of testing requirements for certification procedures.

In addition to applying the technology to aerospace propulsion structures (including SSME) by Rocketdyne, the NESSUS technology is general and has been applied by SwRI, NASA, and many industries to various engineering systems that include automotive, aircraft, offshore, geomechanics, biomechanics, and other structural systems.

In summary, the PSAM project not only has met it's objective, but has made a major impact on the development of reliability technology and played a major role in promoting the widespread use of probabilistic analysis and design technology.

Based on the PSAM technology, SwRI team has conducted PSAM technology transfers through its annual short course since 1990 and on-site short courses at industrial companies. More than two hundred engineers have been trained through these courses. Based on this experience, the following future work is recommended:

### **Technology Transfer**

Design and develop two short courses: 1) PSAM theory and applications, and 2) NESSUS training.

### Improve NESSUS user-interface

A graphical interface will significantly reduce the training time and encourage more users. This interface should help the users develop NESSUS input without extensive probabilistic background or manual reading and present important probabilistic results (such as sensitivity charts, probabilistic stress field) graphically.

### Enhance NESSUS reliability-based design capability

Integrate NESSUS with a design optimization tool to automate the reliability-based design process. This will significantly reduce the engineer's and computer time to perform reliability-based design optimization. This capability will also be useful for optimizing cost, weight, inspection schedule, and other decision parameters.

### Enhance probabilistic analysis algorithm

An extremely valuable capability can be developed by implementing an intelligent adaptive probabilistic algorithm that will automatically switch from one method (e.g., AMV) to another (e.g., adaptive importance sampling) for complicated problems.

### Enhance NESSUS code architecture for parallel processing

Probabilistic analysis requires multiple independent deterministic analyses. These analyses are well suited to parallel processing. Parallel processing using multiple computers has the potential to improve significantly the efficiency of the probabilistic analysis.

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13. ABSTRACT (Maximum 200 words)			
<p>Probabilistic Structural Analysis Methods (PSAM) are described for the probabilistic structural analysis of engine components for current and future space propulsion systems. Components for these systems are subjected to stochastic thermomechanical launch loads. Uncertainties or randomness also occurs in material properties, structural geometry, and boundary conditions. Material property stochasticity, such as in modulus of elasticity or yield strength, exists in every structure and is a consequence of variations in material composition and manufacturing processes. Procedures are outlined for computing the probabilistic structural response or reliability of the structural components. The response variables include static or dynamic deflections, strains, and stresses at one or several locations, natural frequencies, fatigue or creep life, etc. Sample cases illustrates how the PSAM methods and codes simulate input uncertainties and compute probabilistic response or reliability using a finite element model with probabilistic methods.</p>			
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